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THE COSMIC ABUNDANCES OF POTASSIUM, URANIUM, AND THORIUM AND THE HEAT BALANCES OF THE EARTH, THE MOON, AND MARS

By Harold C. Urey

ENRICO FERMI INSTITUTE FOR NUCLEAR STUDIES, UNIVERSITY OF CHICAGO, AND CLARENDON LABORATORY, OXFORD, ENGLAND (1956–1957)

Communicated October 23, 1956

Under the above title the writer published a paper (Urey, 1955)[1] in which an incorrect assumption was made. It is the purpose of this note to call attention to this and to make what appears at the present time to be a correct, or more nearly correct, solution of the problem.

The problem of the amounts of the radioactive elements potassium, uranium, and thorium has been a troublesome one for many years. The abundance of potassium in the meteorites has been solved satisfactorily only very recently (Ahrens, Pinson, and Kearns, 1952; Edwards, 1955; Edwards and Urey, 1955).[2] The amounts of potassium in the chondritic meteorites is remarkably constant, and the assumption that the chondritic meteorites give a reasonable sample of "cosmic" and "solar" nonvolatile matter proves to be a very reasonable hypothesis (Suess and Urey, 1956).[3] An average of Edwards' results, which are the most extensive and appear to be the most reliable, gives about 823 p.p.m. of potassium. This is slightly less than the 880 p.p.m. used previously. The achondrites contain less potassium than the chondrites, and some weight should be given to them. The uncertainty in the datum is surely greater than the difference in these two values.

A very great variability in the analytical data for uranium in chondritic meteorites exists in the literature. Davis (1950)[4] determined uranium in the Shaw and Cumberland achondrites as 4.8 and 1.0×10^{-8} gm/gm, respectively. However, the chemical composition of achondrites is very variable, and the mean uranium content of solar matter cannot be deduced from these data.

Tilton (1951)[5] found 1.1×10^{-8} gm/gm in the Modoc chondrite, but both he and his co-workers have been far from confident that the result was reliable.

Very sensitive procedures have been developed by Chackett and co-workers (1950),[6] and these have been applied to the Beddgelert chondrite. The uranium and thorium contents were reported as 0.106 and 0.335×10^{-8} gm/gm, respectively. These values were secured by averaging their results using approximately 17 per cent of metal by weight. Patterson (1955)[7] has isolated lead from the iron meteorites Canyon Diablo and Henbury and from the Modoc and Forest City chondrites and has determined their isotopic compositions and the amounts of

lead in the chondrites. These data are sufficient to fix the age since the lead of the iron meteorites became separated from uranium and to determine the amounts of uranium necessary to produce the observed radiogenic leads of the chondrites. Patterson's results on uranium abundances agree with the data of Chackett *et al.*

All these recent data appear to have been obtained very carefully, and yet, if all are correct, the uranium contents of chondritic meteorites vary very greatly, and this is not true for other elements which have been determined by reliable methods. It was and is my belief that all these data are not correct, and in this case an average is not correct either. In the previous article I selected the data of Chackett *et al.*, which are essentially the same as those of Patterson. Others have used averages or have selected the data of Davis and Tilton.

Recently Turkevich, Hamaguchi, and Reed (1956)[8] have developed the neutron activation method for determining uranium. In this method the sample is exposed to neutrons before any chemical processing is used. Np^{239} is produced from U^{238}, and radiogenic lanthanum and barium are produced from U^{235}. Subsequent chemical separations are directed to the isolation of neptunium and radio barium. These do not occur in nature, and hence contamination is impossible. Tracers were used to correct for losses. Thus the great sources of error inherent in previous procedures are avoided. Their results agree with the low values of Tilton rather than with the higher values of Chackett *et al.*, and of Patterson. The data for the Forest City, Modoc, Richardton, and Holbrook chondrites vary from 0.89 to 1.51, with an average of 1.13, all $\times 10^{-8}$ gm/gm. This variation is due partly to errors of the analytical method. Thus the constancy of composition of the chondrites is indicated again. One concludes that the data of Chackett *et al.* are too high or that Beddgelert is a most unusual chondrite. Also, the amounts of uranium in Modoc and Forest City are insufficient to produce Patteron's radiogenic leads. It appears that he must have had some contamination from his reagents. It should be noted that it is very difficult to avoid contamination by lead from reagents when the concentrations of the element are so low, because lead is a commonly used metal in commercial chemical apparatus.

These new data solve the difficulties which were the primary concern of the previous article. The thorium abundance has not been determined as yet but presumably is 3 or 3.5 times that of uranium. Taking 823 p.p.m., 1.13×10^{-8} gm/gm, and 3.8×10^{-8} gm/gm for the potassium, uranium, and thorium concentrations, respectively, for the chondritic meteorites and assuming the same concentrations for the earth, moon, and Mars, no serious difficulties in heat balances occur. It is possible that the moon was formed at low temperatures and has never melted, that Mars was formed at low temperatures and has never formed a core, and that the rate of loss of heat from the earth's interior is adequate to remove all the radiogenic heat. This problem, which has been a continuing concern for the writer for some years, is solved by these new data, and the arbitrary assumption that the radioelements were concentrated in the meteorites by some process by a factor of 3.19 is unnecessary. The heats generated in 4.5×10^9 years are rather too high for a cold moon, but we still do not know heat conductivities at all well, and other constants are not too certain.

Table 2′-4′, which is a recalculation of the data given in Tables 2 and 4 of the previous paper, gives the results of calculations on meteoritic matter using the

new concentrations. The rate of heat generation is now 1.57 ergs per year and gram as compared to an average of 2.2 ergs per year and gram required by the observed heat loss of the earth. Thus the rate of generation of heat by radioactive elements, assuming that the earth has the mean composition of the meteorites with respect to the radioactive elements, is less than the rate of heat loss observed. This is to be expected if the earth's rate of heat generation is decreasing with time.

TABLE 2'-4'

HEAT PRODUCTION IN CHONDRITIC METEORITES

(K = 823 p.p.m.; U = 1.13 × 10^{-8} gm/gm; Th = 3.8 × 10^{-8} gm/gm)

NUCLIDE	dE/dt AT PRESENT (ERGS/YR GR OF METEORITE)	TOTAL E IN JOULES PER GRAM OF CHONDRITIC METEORITE IN THE LAST 5×10^8 YEARS	IN THE LAST 4.5×10^9 YEARS	IN THE LAST 5×10^9 YEARS	IN THE LAST 5.5×10^9 YEARS	FROM 5.5 TO 4.5×10^9 YEARS AGO
K^{40}	0.92	52	1,790	2,400	3,210	1,420
Th^{232}	0.31	16	159	178	199	40
U^{234}	0.01	1	118	189	313	195
U^{238}	0.33	17	217	253	290	73
Total	1.57	86	2,284	3,020	4,012	1,728

It is also evident that the radioactive heat generation during remote times, though much greater than at present, was insufficient to produce general melting of well-insulated solid objects. The melting, which produced the silicate minerals and the kamacite and taenite of the chondritic meteorites, must have occurred before $\sim 4.3 \times 10^9$ years ago as shown by the K^{10}-A^{40} age determinations. Thus it seems most improbable that radioactive heating due to these elements could have been more than a minor factor in the melting of the minerals of the chondritic meteorites.

Many details of the previous discussion must be modified as a result of these new data, but I believe no serious errors in understanding the paper will be made by the reader if the corrections presented in this note are kept in mind.

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[2] L. H. Ahrens, W. H. Pinson, and M. M. Kearns, 1952; *Geochim. et cosmochim. acta*, **2**, 229, G. Edwards, *Geochim. et Cosmochim. Acta*, **8**, 285, 1955; G. Edwards and H. C. Urey, *Geochim. et cosmochim. acta*, **7**, 154, 1955.

[3] H. E. Suess and H. C. Urey, *Revs. Mod. Phys.*, **28**, 53, 1956.

[4] G. L. Davis, *Am. J. Sci.*, **248**, 107, 1950.

[5] G. Tilton, (1951), dissertation, University of Chicago, published in C. Patterson, H. Brown, G. Tilton, and M. Inghram, *Phys. Rev.*, **42**, 1234, 1953.

[6] K. F. Chackett, J. Golden, E. R. Mercer, F. A. Paneth, and P. Reasbeck. *Geochim. et cosmochim. acta*, **1**, 3, 1950.

[7] C. Patterson, *Geochim. et cosmochim. acta*, **7**, 151, 1955, and references given there.

[8] A. Turkervich, H. Hamaguchi, and G. Read, private communication, 1956, and Gordon Research Conference, summer, 1956.

*THE ISOLATION OF A NEW VIRUS ASSOCIATED WITH RESPIRATORY CLINICAL DISEASE IN HUMANS**

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Communicated by K. F. Maxcy, March 19, 1956

The isolation of the RI-APC group of respiratory viruses[1, 2] through the use of tissue culture methods has greatly stimulated interest in this field.

In this paper we wish to report the isolation of a new respiratory virus (JH), which from all available data is responsible for a clinical disease in the human population.

RESULTS

Isolation of Virus.—Naso-pharyngeal washings were collected in monkey kidney maintenance media. They were shell frozen in ampules in an alcohol–dry ice mixture and stored at −50° C. When isolation attempts were made the ampule was quickly thawed and 0.1 ml. of the washing was inoculated into tubes of Hela cells as described by Hilleman and Werner.[1] Our methods follow theirs exactly except the maintenance media of Eagle[3] was substituted. The virus which will be described in this paper did not cause degeneration of the Hela cells within 240 hours, even after 3 blind passages. The twenty isolations on which most of this work has been done elicited titers of approximately 10^6 to 10^7 tissue culture infecting doses per 1.0 ml. of the infected monkey kidney cells. The agent multiplies in monkey kidney cells grown in 3 per cent inactivated horse serum, 5 per cent beef embryo extract, and 92 per cent 199 solution. This media contains 100 units of penicillin and streptomycin per one ml. of media. Monkey kidney cells are destroyed by the virus.

The cytopathogenic effect of this agent can usually be observed more readily in 199 medium without the above additions. However, in a few primary isolations this virus was isolated in the richer medium and not in plain 199 medium. The significance of these observations is not clear at the present time

Epidemiological and Clinical Data.—Although detailed epidemiological and clinical data will be reported elsewhere, brief mention will be made here of certain of the data.

The virus has been isolated from human respiratory cases in all seasons of the year from both adults and children. It has been isolated from two large respiratory outbreaks, as well as from individuals clinically ill with respiratory infections who came to the Johns Hopkins Hospital or to the Baltimore city hospitals for treatment. The main clinical features with which this agent is associated are malaise and coryza and mild sore throat. A low-grade fever may or may not be present. There are no physical findings to suggest lower respiratory involvement. The duration of clinical symptoms averages about three days. Bacteriological analysis of the throat and nasopharynx swabs of individuals ill with the above syndrome show normal flora. Antibody surveys on various age groups carried out so far have shown that this virus is relatively widespread, about 20 per cent of the indi-

viduals over eight years of age showing the presence of neutralizing antibodies. Complement-fixing antibodies also can be shown to be present for this agent in the sera of individuals.

Serological Characteristics of Etiological Agent.—Table 1 shows a typical neutralization test using acute-convalescent sera of individuals from whom the virus has been isolated who were clinically ill with the symptoms described above. It can be readily seen that there is a large increase in the neutralizing antibody level.

TABLE 1

NEUTRALIZATION TESTS ON ACUTE-CONVALESCENT SERA OF PATIENTS FROM WHOM THE JH AGENT WAS ISOLATED*

Patient	Acute Titer	Convalescent Titer	Patient	Acute Titer	Convalescent Titer
M. W.	<1:2	1:64	J. S.	<1:2	1:64
D. L.	<1:2	1:64	I. A.	<1:2	1:32
H. S.	<1:2	1:128			

* All convalescent sera were collected about 4 to 6 weeks after the acute sample was taken.

TABLE 2

RELATION OF DIFFERENT DISEASES AND ANTIBODY RISE TO JH AGENT

Infection	No. Cases Tested	Immunologic Type	Acute-Convalescent Rise in Neutralizing Antibodies*
Influenza	18	A′	0
Influenza	15	B	0
Influenza	4	C	0
RI-APC	10	3	0
RI-APC	5	4	0
Scarlet fever	8	Group A, Types 44 & 22	0
Streptococcal sore throat	10	Group A, Types	0
Poliomyelitis	5	1	0
Poliomyelitis	5	2	0
Poliomyelitis	5	3	0
Q fever	10	...	0
Rocky Mountain spotted fever	10	...	0
Murine typhus	5	...	...
Rheumatic fever	11	...	0
Tuberculosis	6	...	0
Infectious mononucleosis	8	...	0
"Common cold"	16	...	0
Mumps	10	...	0
Infectious hepatitis	6	...	0
Herpangina	8	...	0
Primary atypical pneumonia with rise in cold agglutinins	10	...	0
Nonbacterial meningitis	12	...	0
Measles	7	...	0
Rheumatoid arthritis	8	...	0
Coronary thrombosis	10	...	0

* A four-fold rise was considered significant.

These tests were carried out as described above using 10 to 50 TCD_{50}. All isolations of the agent seem to be immunologically similar as determined by neutralization tests.

Table 2 shows that there was no significant increase in titer against this virus in acute-convalescent sera of the following infections: influenza, RI-APC, Q

fever, Rocky Mountain spotted fever, scarlet fever, streptococcal sore throat, rheumatic fever, and poliomyelitis. All these sera showed very high increases in titer in the convalescent sera against the homologous etiological agents. Furthermore, acute-convalescent sera from various other diseases shown in Table 2 showed no increase in neutralizing titer in the convalescent sera to the JH agent. All these data indicate that the increase in neutralizing titer is specific.

No complement-fixing (CF) antigen could be prepared from monkey cells infected with the JH agent which reacted with standard RI-APC sera using methods which could readily detect RI-APC CF antigen in Hela cells infected with a RI-APC virus.[1] The JH preparation used for these CF experiments titered out to approximately 10^6 tissue-culture-infecting doses. Standard sera to RI-APC types 1–7 which neutralized 1,000 tissue-culture-destroying doses of these viruses did not neutralize 10 tissue-culture-destroying doses of the JH agents, all tests being carried out in monkey cells under the same conditions.

The acute-convalescent sera of the individuals shown in Table 1 did not show any hemagglutination inhibition rise to influenza A (PR8), influenza A′ (FW-1-50), influenza B (B-1-11), influenza C (strain 1233), or a strain of swine influenza. None of the sera showed rises to the RI-APC CF antigen supplied by Microbiological Associates.[4] None of the convalescent sera showed evidence of cold or streptococcus MG agglutinins, or a rise in streptolysin O antibodies.

Other Characteristics of the JH Agent.—The agent can be passed in series in monkey cells without loss of titer at least 10 times, after which no further attempts were made to pass the agent. When the JH suspension, which titered about 10^{-5} in monkey cells, was inoculated intranasally, intracerebrally, or intraperitoneally into twenty 1-day-old mice or ten 10-day-old hamsters, no overt signs of disease were noted over the 21-day observation period. Similar negative results were obtained when ten 15–17-gm. mice or six young adult guinea pigs were inoculated intraperitoneally. The baby mice showed no lesions in their thoracic or abdominal viscera when some were sacrificed 7 or 12 days following inoculation. Intranasal inoculation of ferrets gave negative results, the animals showing normal fever curves and showing no overt signs of illness. Two blind passages of various organs and tissues under all the above conditions failed to show any evidence that the JH agent had multiplied.

Inoculation of the JH suspensions which contained about 10^2 and 10^5 TCD_{50} into 12-day-old chick embryos by the amniotic and allantoic routes failed to reveal evidence of multiplication even after three blind passages, the fluids being harvested 72 hours after inoculation and titered in monkey cells. The amniotic and allantoic fluids did not agglutinate chicken, human "O", or guinea pig red cells at room temperature or at 4° C.

The JH virus does not appear related to the ECHO group of viruses.[5] The JH virus shows a very different cytopathogenic effect than the ECHO viruses in monkey kidney cells. It is not neutralized by antisera prepared to ECHO viruses 1, 2, 3, 6, 5, 7, 8, and 9. The JH virus also has very peculiar nutritional requirements which are different from those reported for any ECHO virus.

The JH virus is not neutralized by antisera prepared to Coxsackie A types 1, 2, 5, 6, 8, 9, 10 or B types 1, 2, 3, 4 or 5. The cytopathogenic effect of the JH agent in monkey kidney cells is also different from that associated with Coxsackie viruses.

Attempts To Grow JH Agent on Artificial Medium.—Infected monkey tissue culture preparations which titered out to approximately 10^{-5} in monkey cells were inoculated into Brewers' thioglycolate medium and sheep, human, and rabbit blood agar plates. These cultures were inoculated both aerobically and anaerobically, and none showed growth after 12 days at 30° C. The JH agent could be filtered through standard bacteriological sterile filters which completely removed all *Staphylococcus aureus* present in prepared suspensions.

DISCUSSION

The data presented in this paper indicate that the agent described is a new virus responsible for clinical illness in the civilian population. This statement is based on the following facts: (1) the isolation of the agent from individuals ill with a particular respiratory clinical syndrome; (2) the rise in antibodies to the agent in such people and the failure to show a rise in antibody to the agent in many other human respiratory as well as other viral, bacterial, rickettsial, and chronic diseases; (3) the failure to isolate the agent from 1,100 individuals not showing clinical signs of respiratory illness in the same seasons and from the same age group and in the same state as the ill individuals from whom the virus was isolated;[6] (4) the relationship between antibody level and clinical disease in certain respiratory outbreaks. Thus, in one large respiratory outbreak where the JH virus was isolated from many sick individuals, there was a definite correlation between the individuals' blood sera neutralizing antibody titers to the JH virus one month previous to the outbreak and their developing the clinical signs of respiratory infection during the epidemic. The higher the titer to the JH virus previous to the respiratory outbreak, the less chance a person had of developing overt illness. No such relationship was found for influenza A′ or RI-APC antibodies. Furthermore, 16 individuals who developed respiratory infections during this epidemic did not show the presence of the JH virus in their nasal pharyngeal washings taken one month previous to their respiratory infections. However, from 11 of these individuals the JH virus was isolated from their nasal pharyngeal washings when they were ill during the respiratory outbreak, 10 of the 11 individuals showing significant rises in neutralizing antibodies to the JH virus. Such longitudinal studies as these where the same individuals are studied for a long period of time with test samples being taken at frequent intervals, greatly help in deciding whether a virus is the cause of a disease.

The viruses responsible for the major cause of respiratory disease in the civilian population have not as yet been identified. In our extensive field-laboratory study over the last three years, less than 30 per cent of the respiratory illnesses could be identified as being caused by a known microparasite, of either bacterial or viral origin.[6]

The importance of the JH agent in causing illness in the civilian population as well as other problems dealing with this virus are under investigation and will be described elsewhere. The virus seems fairly widespread, as our surveys carried out so far indicate that about 20 per cent of the population over eight years of age have antibodies to this agent.

Clinically, the virus described in this paper is associated with a very mild upper respiratory infection. The major difference between this illness and the common

cold is that the JH virus is usually associated with low grade fever, as well as slight malaise, coryza, mild sore throat, and in some cases a cough. However, in view of the similarity between the mild respiratory infection associated with the JH virus and the common cold we hope that study of the JH virus may lead to some clues as to how the common cold virus may be isolated. In this connection it should be pointed out that the JH virus has a very long incubation period before producing a cytopathogenic effect in monkey kidney cells, particularly on primary isolation. Thus, a period of 25 days, as well as a blind passage at this time, has been found to be required in many of the isolations, the cytopathogenic effect showing up on the tenth to sixteenth day of the second passage.

* This work was supported by a grant from the Rockefeller Foundation.

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[4] This CF antigen will react with all types of RI-APC viruses and is supplied by Microbiological Associates, Bethesda, Maryland.

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REVERSIBILITY AND EQUILIBRIUM OF THE GLUTAMINASE REACTION OBSERVED CALORIMETRICALLY TO FIND THE FREE ENERGY OF ADENOSINE TRIPHOSPHATE HYDROLYSIS

By T. H. Benzinger and R. Hems

Communicated by S. B. Hendricks, September 28, 1956

UNITED STATES NAVAL MEDICAL RESEARCH INSTITUTE, BETHESDA, MARYLAND, AND MEDICAL RESEARCH COUNCIL UNIT FOR RESEARCH IN CELL METABOLISM, DEPARTMENT OF BIOCHEMISTRY, UNIVERSITY OF OXFORD, ENGLAND

The enzymatic hydrolysis of glutamine in the absence of adenosine phosphates is considered a practically irreversible[1] reaction. Attempts have therefore been made by Levintow, Meister, and Morales[2] to derive its standard free-energy change, $\Delta F'^{\circ}$, indirectly, from available thermal data and "third-law entropies" of asparagine and ammonium aspartate. Although the argument of analogy with asparagine has recently been shown to be invalid,[3] the biological significance of thermodynamic data on glutamine hydrolysis is obvious: Elliott's glutamine synthetase reaction,[4] in which glutamine synthesis is coupled with the hydrolysis of adenosine triphosphate (ATP) has a well-measurable ratio of reactants and products at equilibrium,[2] and the difference between the free energies of the glutaminase and glutamine synthetase reactions is the free energy of ATP hydrolysis.

In an effort to detect with glutaminase the reverse process, glutamine synthesis, and to measure directly a possible glutaminase equilibrium, we have employed a method recently described in these Proceedings:[5] Free-energy, heat, and entropy changes of equilibrium reactions may be obtained from two calorimetric measurements.

C. Kitzinger and R. Hems[6] have shown experimentally with the reversible enzymatic conversion of fumarate into malate that 16 cc. each of M/1,000 solutions of reactant and product suffice to carry out this determination on reactions of reasonable heat change and reasonably low change of free energy. For an almost irreversible reaction, such as glutamine hydrolysis, where Q_1 [cal/mole] (the heat change observed upon addition of enzyme to the reactants) approaches ΔH, while Q_2 (the heat change observed upon addition of enzyme to the products) approaches zero, a modification of the calorimetric procedure was required. Only with a concentrated solution of products and with a pure and powerful enzyme preparation could we expect to observe the heat change Q_2, associated with the reverse process. However, once the reverse process had been observed, the equilibrium ratios of concentrations of reactant and products were readily found by addition of varying small quantities of the reactant to the concentrated product solution and subsequent incubation with enzyme in the microcalorimeter.

The enzyme, glutaminase, was prepared from *Clostridium welchii*, according to Hughes and Williamson,[7] and further purified by additional safranin precipitation, then freed of low-molecular contaminants by dialysis, and concentrated by freeze-drying. One hundred and sixty microliters of this preparation were required to carry the reaction to the equilibrium point in the microcalorimeter within one hour, transforming 12 micromoles of glutamine into ammonium glutamate, or vice versa.

In Figure 1, *a*, *b*, *c*, three original recordings of heat changes are reproduced and explained. It appears that a solution approximately 1 molar in ammonium glutamate and 1/1,000 molar in glutamine neither produces nor absorbs substantial heat when incubated with the enzyme (Fig. 1, *b*). Here reactants and products are present at nearly the equilibrium ratio of their concentrations, whereas, with every other ratio, either glutamine synthesis absorbing heat (Fig. 1, *a*) or glutamine hydrolysis producing heat (Fig. 1, *c*) takes place.

Using in a series of such tests equal quantities of the same enzyme and plotting total calories absorbed or, even more simply, rates of reaction (Fig. 2) taken at the same time after mixing against the initial ratios of concentrations [glutamate$^-$]$\times$ [NH_4^+]/[glutamine] one finds the intersect with zero heat change, that is, the equilibrium point, with ease and precision. It appears that, over the intentionally narrow range of our observations, reaction rates are proportional to the distance from equilibrium and that the rates are identical for the back and forward reactions, as they should be at the equilibrium point. Table 1 shows the results of different experiments, as evaluated graphically.

In Table 2 thermodynamic equilibria and free energies are reported for various concentrations of the reactants. T. Spencer and K. Bascombe have calculated and permitted us to use here and in a forthcoming paper[8] activity coefficients of ammonium glutamate (see Kielland[11]). The data reported as a result of our study depend upon the assumption that these coefficients are correct. They were considered the best available, although we were unable to substantiate them by extrapolation from the measurements which Hoskins, Randall, and Schmidt[12] have obtained at much lower concentrations of ammonium glutamate. The activity of glutamine in 0.9M ammonium glutamate solution was found not to be significantly

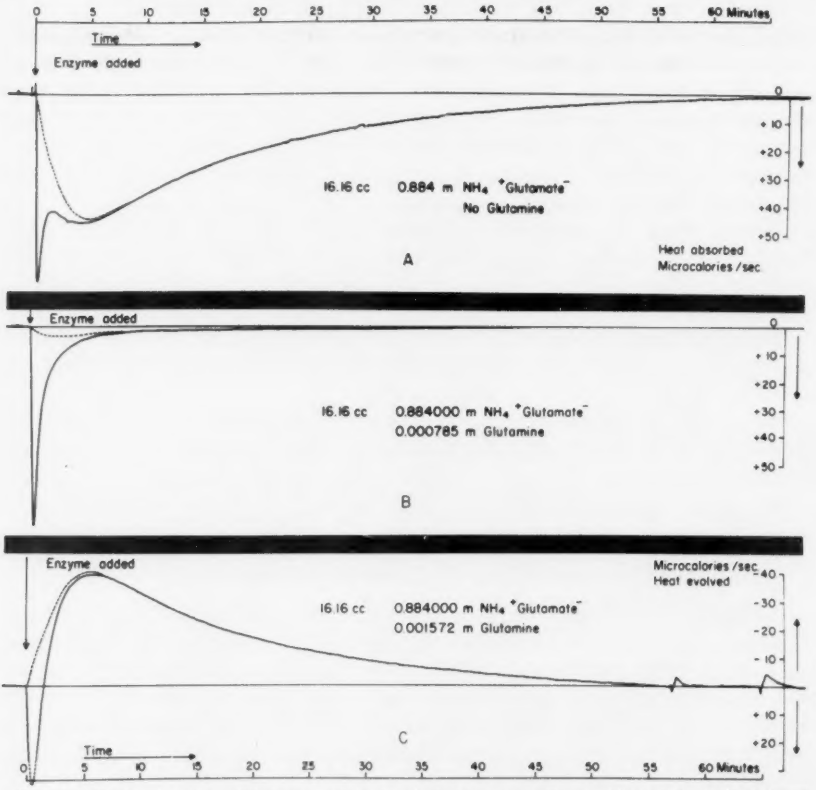

FIG. 1.—Glutaminase equilibrium; original calorimeter recordings. Except for initial burst of heat absorption caused by dilution of salts in enzyme drop, mixture of 0.88 M NH_4 glutamate with 0.78×10^{-3} M glutamine (B) neither produces nor absorbs substantial heat. In pure glutamate, glutamine synthesis (associated with heat absorption) takes place (A). In 0.88 M glutamate with 1.57×10^{-3} glutamine, glutamine hydrolysis (associated with heat production) is observed (C).

TABLE 1

GLUTAMINASE EQUILIBRIUM BY MICROCALORIMETRY*

Experimental Series	Glutamine Required for Equilibrium (μMoles)	Glutamine Final Molarity	Glutamate$^-$ NH_4^+ Final Molarity	$\frac{[\text{Glutamate}^-] \times [NH_4^+]}{[\text{Glutamine}]}$ at Equilibrium (Per Cent)
I (6 experiments)	13.35 ± 0.1	0.8250×10^{-3}	0.884	950 ± 1%
II (3 experiments)	3.86 ± 0.1	0.2385×10^{-3}	0.444	826 ± 2.5%
III (3 experiments)	1.035 ± 0.1	0.0644×10^{-3}	0.223	773 ± 10%

* Molar equilibrium ratios of reactants and products of the glutaminase reaction at various concentrations. Activity coefficients cause the differences shown in the last column. Standard deviations include only calorimetric errors.

different from unity, by calorimetric measurements of solubility, at the suggestion of K. Burton. The standard free-energy change of glutamine hydrolysis observed is $\Delta F^\circ_{298} = -3{,}500$ cal/mole. It appears from the linear relationship shown in

Figure 2 that two calorimetric measurements would usually suffice to find the equilibrium constant. The heat change, ΔH, is best determined with dilute solutions of glutamine. Its measurement, leading to $\Delta H = -5{,}300$ and $\Delta F'^{\circ}_{310} = -3{,}430$ cal/mole, will be described elsewhere,[3] since it was not essentially different from similar determinations with other irreversible reactions, such as ATP hydrolysis[9] or ribulose diphosphate carboxylation.[10] A more detailed account of the present study is in preparation.[8]

Discussion.—The question may be raised whether it is permissible to attribute the heat changes observed to glutamine synthesis, without a chemical identification of any of the material practicles involved. The answer is that only a process with kinetics identical to glutamine hydrolysis and opposite sign of the heat change could have produced the pictures shown in Figure 1, *a* and *c*, and caused the heat change to be practically zero at all times of observation in Figure 1, *b*. The exist-

TABLE 2

FREE ENERGY FROM GLUTAMINASE EQUILIBRIUM*

Experimental Series	NH_4^+ and Glutamate$^-$ Molarity	Activity Coefficient γ^2	$\frac{[\text{Glutamate}^-] \times [NH_4^+]}{[\text{Glutamine}]}$ at Equilibrium	Thermodynamic Equilibrium Constant	Temperature (° K.)	Free Energy (cal/Mole)
I	0.884	0.648	950 ± 1%	399	297.4	−3,520
II	.444	.675	826 ± 2.5%	377	298.0	−3,510
III	0.223	0.730	773 ± 10%	412	298.0	−3,560

*Free energy of the glutaminase reaction obtained with activity coefficients calculated by T. Spencer and K. Bascombe, based on the formula of Kielland, *J.A.C.S.*, **59**, 1675, 1937.

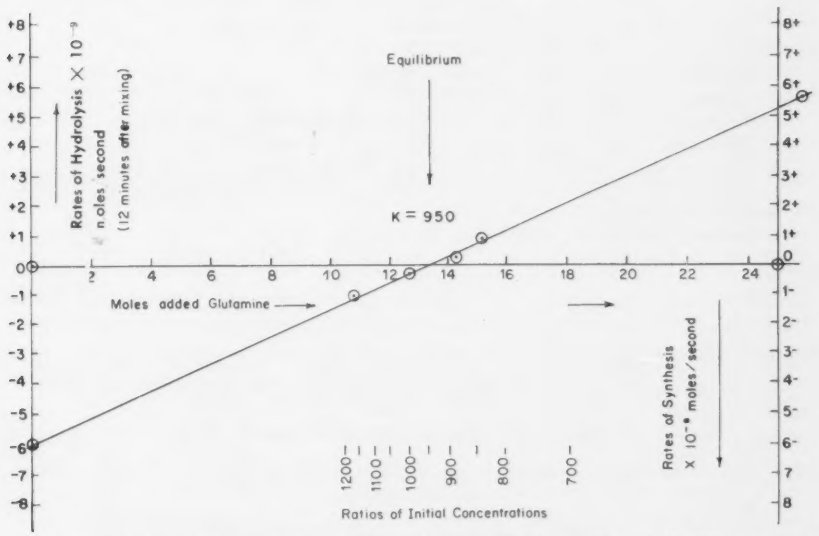

FIG. 2.—Calorimetric determination of glutaminase equilibrium. To a 0.884 *M* solution of ammonium glutamate, varying small quantities of glutamine were added, and the mixtures were incubated with the enzyme glutaminase in the dissipation microcalorimeter. Rates of glutamine hydrolysis (associated with heat production) or glutamine synthesis (associated with heat absorption) are plotted against quantities of glutamine initially present. The extrapolated intersect with zero heat change is the molar equilibrium ratio, 950.

ence of such a reaction is, of course, most improbable. Interaction of the product, glutamate, at low concentration with the enzyme used had been shown by calorimetry to be absent, and full activity of the enzyme in such tests as Figure 1, *b*, was ascertained by addition of glutamine to the sample in a second calorimetric test after the equilibrium run.

In summary, it has been shown experimentally that the calorimetric determination of free-energy, heat, and entropy changes as outlined in these PROCEEDINGS[5] may be applied to a seemingly irreversible reaction. The standard free-energy change of glutamine hydrolysis observed is $\Delta F'^{\circ}_{310} = -3{,}430$ cal/mole. The sum of this quantity and whatever value is accepted for the free energy of the glutamine synthetase reaction is the free energy of ATP hydrolysis. For one example, with the value of $-4{,}300$ cal/mole at pH 7.0 and 37° C.,[2] the free energy of ATP hydrolysis is $\Delta F' = -7{,}730$ cal/mole, the heat change $\Delta H = -4{,}800$ cal,[10] and the standard entropy change $\Delta S' = +9.45$ e.u.

It is our pleasant obligation to thank Professor H. A. Krebs, in whose department these experiments were carried out with the microcalorimeter of the Bio-Energetics Division, Naval Medical Research Institute, for continuous encouragement and advice. Thanks are also due to Drs. T. Spencer and K. Bascombe, of the Department of Physical Chemistry, Oxford, for their calculation of activity coefficients used in this paper and to Dr. K. Burton for valuable advice and criticism.

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*ON THE FUNCTIONS OF X-IRRADIATED "FEEDER" CELLS IN SUPPORTING GROWTH OF SINGLE MAMMALIAN CELLS**

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In earlier publications we have described a method whereby suspensions of single mammalian cells can be plated in Petri dishes containing nutrient medium, so that each cell multiplies in isolation to form a macroscopic colony.[1, 2] While some cell

lines were found to give rapidly multiplying colonies with almost 100 per cent plating efficiency on plates containing only the standard nutrient solution, others grew poorly or not at all in the media available unless the plate was also inoculated with a "feeder" cell layer, consisting of about 10^5 cells previously exposed to a completely lethal dose of X-irradiation.[2, 3] In the presence of such a "feeder" layer of nonmultiplying cells, the test cells exhibited maximal plating efficiencies and growth rates. Two different mutants (S1 and S3) of the HeLa cell, of human carcinomatous origin, have been isolated and quantitatively characterized on the basis of the need of one of these (S1), but not the other (S3), for assistance by a feeder layer order for growth to occur in a specific medium.[3]

The experiments here to be described were undertaken to clarify the mechanisms by which a feeder layer of nonmultiplying cells can operate to support growth of single cells in an otherwise incompetent medium.

METHODS AND MATERIALS

Specific details of the preparation of monodisperse cell suspensions, their plating in Petri dishes and incubation in an atmosphere of 5 per cent CO_2, have been previously recounted.[2, 4] All plates, unless otherwise indicated, received 4.5 cc. of the Complete Growth Medium, as previously described (Table I of another paper[4]), except that the horse serum was omitted and the amount of human serum was often varied between the values of 8 and 20 per cent, without significant change in the results. Other specific changes in the medium, like the production of inositol deficiency or the addition of specific antisera, are described in connection with the appropriate experiments. No agar was employed in any of the present platings. The clonal HeLa strain, S3, which had been isolated from the parental HeLa cell population[3] was used for these experiments.

Where feeder cells were utilized, these were prepared by exposure to an X-ray dose of 2,000 r.[2] To each feeder plate, 10^5 of these cells, washed to eliminate carry-over of nutrients, and the new test medium were added before addition of the inoculum of test cells. The ability of this X-ray dose completely to prevent colony formation by the feeder cells themselves has been thoroughly demonstrated.[2, 5]

The cells were incubated for 9–12 days. Plates containing feeder cells sometimes were given a change of medium after 6 or 7 days of incubation if the pH fell sufficiently to indicate depletion of nutrients.

EXPERIMENTAL RESULTS

Two obvious mechanisms by which a feeder layer might operate to promote the growth of single test cells are (1) by direct supply of essential molecules which are absent from the medium and (2) by removal of inhibitory substances which might be present. Experiments were designed with model systems to determine whether a feeder layer can indeed act in either or both of these capacities.

1. *Test of the Ability of Feeder Cells To Relieve a Nutritional Deficiency of the Medium.*—Test of this first mode of action of a feeder cell layer was carried out by plating cells of the S3 HeLa strain with and without a feeder layer in a medium deficient in a known metabolite. The nutrilite chosen for this experiment was inositol, which had been demonstrated by Eagle to be required for the growth of several mammalian cells.[6]

Following Eagle,[6] dialyzed serum was substituted for whole serum in the nutrient solution, in order to eliminate inositol. Eagle had reported[6] that while other mammalian cells under these conditions showed no growth unless inositol was added to

such a medium, the behavior of his HeLa culture was variable. With the single-cell plating technique here employed, the results are always reproducible: No growth whatever of single cells is obtained with an inositol-free medium; addition of inositol (0.4 μg/cc) to such a medium restores the plating efficiency (i.e., the per cent of the single cells plated which grow into macroscopic colonies) close to the theoretical value.

Experiments were then carried out to determine whether addition of a feeder layer of washed, irradiated cells would permit growth of single S3 cells even in the absence of inositol. The results unequivocally demonstrated that the presence of the feeder layer removes the requirement for an external supply of inositol in the growth of single S3 cells. A typical experiment is illustrated in Table 1. In Figure 1 are shown photographs of plates from such an experiment. The conclusion may be drawn that one of the ways in which a feeder layer can function is by relieving a nutritional deficiency of the medium for the growth of single cells.

TABLE 1

REPRESENTATIVE EXPERIMENT DEMONSTRATING THAT A FEEDER LAYER OF X-IRRADIATED AND WASHED S3 CELLS WILL OVERCOME AN INOSITOL DEFICIENCY OF THE NUTRIENT MEDIUM AND SO SUPPORT COLONIAL MULTIPLICATION OF SINGLE, UNIRRADIATED S3 CELLS *

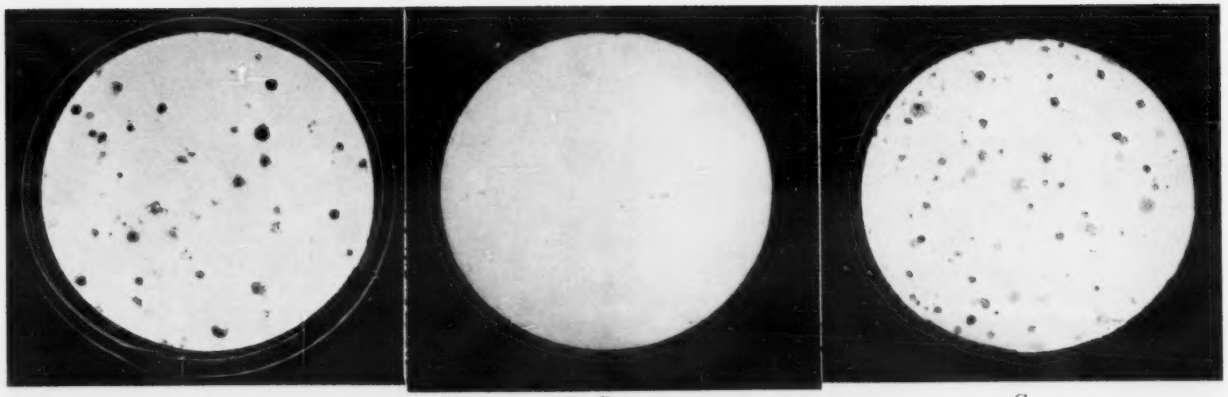
	—PLATING EFFICIENCY (PER CENT)—	
	No Inositol Added	0.4 μg/cc Inositol Added
"Feeders" absent	0	61
"Feeders" present	80	79

*100 washed S3 cells were plated in triplicate in the inositol-free growth medium (standard nutrient solution of Table I of another paper [Marcus, Cieciura, and Puck, *J. Exptl. Med.*, **104**, 615, 1956], 40 per cent; dialyzed human serum, 8 per cent; and Hanks saline, 52 per cent), with and without the addition of inositol and of washed feeder cells, respectively, as indicated. The averaged number of colonies per plate for each set of conditions is equal to the plating efficiency, as given.

2. *Test of the Ability of Feeder Cells To Neutralize a Toxic Medium.*—It has been not infrequently reported that mammalian cell strains are intoxicated by serum obtained from certain human subjects and not from others.[7] It has been presumed that the basis for such specific inhibitory action lies in the presence of antibodies or similar substances in certain sera, capable of preventing growth of some kinds of cells. Hence an experiment was designed to test whether a feeder layer could protect an inoculum of viable cells against the action of toxic antibodies.

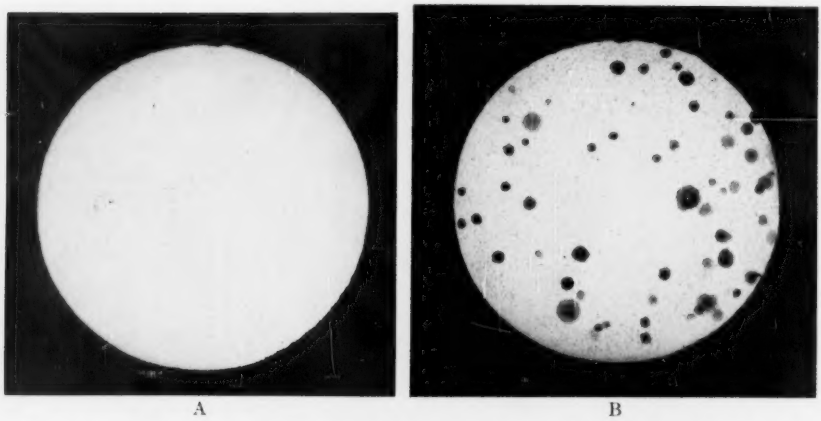
Antibodies to the S3 cell were prepared by intramuscular inoculation of rabbits at weekly intervals with 10^7 washed HeLa cells suspended in 1.0 cc. of Hanks saline. At various time intervals, beginning one week after the third such injection, blood was drawn and serum prepared. Such sera were collected and frozen, and constituted a standard antibody preparation used for all the experiments here presented.

The toxic action of such antibodies for HeLa cells has been described by other investigators.[8] The use of the single-cell plating procedure makes possible precise titration of this toxic antibody action, as will be described elsewhere. An experiment was performed to determine whether the presence of a feeder-cell layer could enable single S3 cells to multiply in the presence of an otherwise inhibitory concentration of antibodies. An inoculum of 100 S3 cells was added (*a*) to a set of plates containing nutrient medium plus a completely toxic concentration of S3 antiserum and (*b*) to an identical set of plates containing, in addition, 10^5 X-irradiated S3 cells as a feeder system. The data of a representative experiment are presented



A B C

FIG. 1.—Typical plates demonstrating that in the presence of an irradiated feeder-cell layer single S3 cells can form colonies in a medium lacking inositol. (Actual size.) *A*, inositol added, no feeders; *B*, no inositol, no feeders; *C*, no inositol, but feeders present. All three plates received an identical inoculum of 100 washed S3 cells in inositol-deficient medium consisting of Standard Nutrient Solution, 40 per cent, dialyzed human serum, 10 per cent, and Hanks solution, 50 per cent. Plate *A* in addition received 0.4 μg./cc of inositol. Plate *B* received no further addition. Plate *C* received 10^5 washed S3 cells which had been irradiated with 2,000 r, to serve as a feeder system. The absence of any colonies on Plate *B* contrasts with the almost theoretical yields obtained on Plates *A* and *C*. The background of gray specks on Plate *C* represents feeder cells which have developed into giants (see Puck and Marcus, *J. Exptl. Med.*, **103,** 653, 1956).



A B

FIG. 2.—Plates demonstrating the effect of a feeder system in permitting cell growth in a medium containing an otherwise toxic amount of antiserum. Both plates received 100 S3 cells in 4.5 cc. of Complete Growth Medium plus 0.8 per cent of S3 antiserum. In Plate *A* the antiserum has completely destroyed the reproductive capacity of the test cells. In Plate *B* the presence of 10^6 irradiated S3 cells has largely neutralized the effect of the antiserum and produced excellent colonial growth.

in Table 2 and demonstrate how the presence of the feeder layer permits maximal plating efficiency and growth rate of single cells, in the presence of antiserum which otherwise completely destroys cell reproductive capacity. Figure 2 presents photographs of plates from such an experiment. Presumably, of course, this detoxifying action of the feeder layer is effected by adsorption of the antibody on the feeder cells.

TABLE 2

DEMONSTRATION OF THE ABILITY OF AN IRRADIATED LAYER OF S3 FEEDER CELLS TO PROTECT AN INOCULUM OF LIVE, SINGLE CELLS FROM AN OTHERWISE TOXIC CONCENTRATION OF S3 ANTISERUM *

Concentration of Added Antiserum (Per Cent)	Feeder Cells (10^5 S3 Cells Irradiated with 2,000 r)	Plating Efficiency (Per Cent)
0	Absent	85
0	Present	80
0.4	Absent	0
0.4	Present	72
0.8	Absent	0
0.8	Present	54

* All plates received Complete Growth Medium plus 10 per cent human serum. In addition, antiserum and feeder cells were added as indicated. Each plate received a live-cell inoculum of 100 S3 cells, which constituted the last addition before incubation.

DISCUSSION

Any of three factors might prevent multiplication of single mammalian cells plated in vitro: (*a*) trauma suffered by the cells in the course of the initial separation from the whole animal or in the dispersal of a cell mass into its individual constituents, (*b*) lack of some essential metabolites, or (*c*) the presence of inhibitory agents in the medium provided. We have elsewhere[2] demonstrated the need for

gentleness in the disaggregation of cells and have shown that quantitative and reproducible growth of each cell plated can be achieved only if conditions are not so drastic as to cause extensive leakage of cell substance into the extracellular fluid. A chemically defined, synthetic medium which will promote unlimited mammalian cell multiplication does not yet exist except for the case of one particular cell.[9] Thus it is necessary to rely on natural sources like serum or embryo extract to supply the as yet unidentified molecular requirements in cultivating most animal cells. However, these complex mixtures usually contain inhibitory substances as well as essential metabolites, and undoubtedly it is the failure to obtain a suitable balance between these which has made it difficult in the past to achieve single-cell growth in high efficiency from a variety of different tissues. We have previously shown that a feeder layer can make such single-cell growth feasible with the same simplicity and high efficiency which is characteristic of microbial systems.[3] The present experiments offer a basis for understanding how such a feeder layer can operate and, indeed, why, as is often the case, single mammalian cells fail to grow under conditions where massive inocula reproduce readily and continuously.[10] The use of a feeder system may make it possible automatically to correct nutritional deficiencies or toxic actions of the biological fluids needed to supplement media in current use. Theoretically, at least, the use of such a feeder system should permit successful single-cell plating of any strain which can at present be grown in massive inocula.

While the use of a feeder system will never replace the need for identification of the complete chemical requirements for growth of each cell strain, it furnishes a powerful tool permitting many kinds of investigation which otherwise could not be carried out at the present time. By making possible single-cell plating, it permits recognition and isolation of mutant markers which can be used as a basis for studies of mammalian cell genetics;[3] more quantitative means of study of cell growth and the effect thereon of various agents (e.g., X-rays)[5] than has before been possible;[11] and more rapid and convenient preparation of clonal cell stocks.

When a single cell has multiplied sufficiently, the resulting colony obviously can perform for itself the functions required initially by the feeder cells. Consequently, the need for the feeders should decrease as growth proceeds.

In this connection, the advantage of using single-cell platings rather than massive inocula for determination of absolute nutritional requirements of mammalian cells becomes obvious. The ability of cells to "feed" each other may mask the presence of certain nutritional requirements. If a mixture of different kinds of nutritional mutants is present in the stock, growth may occur on the addition of only those molecules which are synthesized by none of the cells present. Hence the apparent nutritional requirements may not be complete for any single cell type present in the mixture and may vary significantly for mixtures with different compositions, which may be present in the stocks of different laboratories. Our recent demonstration of the existence of different nutritional mutants in the parental HeLa cell population illustrates the nutritional heterogeneity of such common cell strains.[3]

We have used the feeder technique with human cells originating from a wide variety of noncancerous tissues, including conjunctiva, appendix, liver, kidney, bone marrow, spleen, and skin, as well as several cancerous tissues.[4, 12] In no case have we failed to obtain single-cell multiplication with high plating efficiencies. In many cases it has become possible eventually to dispense with the feeder system,

either because a better medium was ultimately devised or possibly because, in some cases, the cells eventually became sufficiently adapted to the given conditions in vitro to the point where the feeder system was no longer required. It is possible that the feeder system may also be an effective tool for study of cell-cell interaction for specialized cell functions other than reproduction.

Experiments are in progress to determine how wide a variety of human tissues can be successfully plated as single cells, with and without feeder systems, and what kinds of specificities may be involved in the ability of cells of one kind to act as feeders for another in a given medium.

SUMMARY

1. Model studies designed to elucate how a feeder layer of X-irradiated, non-reproducing cells can promote colonial growth of single mammalian cells in an otherwise incompetent medium reveal two ways in which feeder cells can function: They can relieve a specific nutritional deficiency of the medium, and they can neutralize the toxic action of specific cell antibodies.

2. The use of feeder systems as tools for the study of various aspects of mammalian cell behavior is discussed.

* Contribution No. 48. This study has been aided by a grant from the Commonwealth Fund and from the Rockefeller Foundation.

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*THE PRODUCTION OF TUMOROUS ABNORMALITIES IN FERN PROTHALLI BY IONIZING RADIATIONS**

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The appearance and behavior of tumorous growths which occur occasionally in fern prothallial cultures have been discussed both morphologically[1] and cytologically[2] in previous papers. These tumors were considered "spontaneous," in that they appeared relatively rarely and their occurrence could be ascribed to no

known causal factors. In light of their striking similarity in some respects to animal tumors, it was felt that a more intensive study of these tumors was warranted, especially with respect to problems of tumorigenesis. However, the rarity of their occurrence meant that before such studies could be initiated, some means had to be found to increase materially their rate of occurrence.

Ionizing radiations have long been observed to be carcinogenic in animals.[3] More recently, possible similar effects have been observed in plants.[4] Our initial experiments in 1953 indicated that when spores of *Pteridium aquilinum* (the bracken fern) in the dormant condition were given relatively high doses of X-rays, they would subsequently, in sterile culture, produce numerous tumors. Since then, these observations have been conclusively confirmed and extended.

Materials and Methods.—Spores of *P. aquilinum* var. *latiusculum* (Desv.) Underw. were sown in sterile culture in vitro on a simple medium, using the techniques previously described.[1] The X-radiation was administered usually at the rate of 375 r per minute to dry spores and at the rate of 250 r per minute to young prothalli. The gamma radiation was obtained from a cobalt 60 source, with the tubes containing the cultures placed in arcs around it at convenient radii, such that the dosages ranged from approximately 25 to 100 r per day. All experiments were controlled with unirradiated cultures which were otherwise treated like those which were irradiated.

In the experiments in which the dormant, ungerminated spores were irradiated, the controls were taken from the same lot of spores and were sown before the irradiated ones. Doses of X-radiation on dry spores have gone as high as 40,000 r. However, a total dose in the range of 15,000 r has been found to be quite satisfactory in terms of tumor response and convenient in terms of time involved in treatment. Dry spores have also been kept in the gamma-radiation field until they received similar total doses, although over considerable periods of time.

In the experiments on young prothalli, the spores were sterilized and sown on agar slants in test tubes and then allowed to germinate and undergo a few divisions over a period of a week. The controls were then selected at random from these cultures, and the rest of the cultures were given X-radiation, with doses ranging from 250 to 8,000 r. The extent to which the test tube decreased the actual dosage on the prothalli was found to be negligible. In the case of the chronic gamma radiation, the cultures were placed in the radiation field immediately after sowing and were left there for variable periods of time, usually from two to three months. The controls, randomly selected from the initial sowing, were kept under similar conditions of light and temperature in another room.

Results and Discussion.—In each case of either X- or gamma radiation on either ungerminated spores or young prothalli, a dosage range was found at which an unquestionable tumor response would occur. Once the effective range was found, the results were repeatable. After a delay of about a month, tumors become recognizable in slants of young prothalli as nodular masses, usually of a slightly darker green than the surrounding prothalli, which remain primarily filamentous. In several experiments, numbers of nodules suspected to be tumors were isolated and grown in separate culture as a check on the criteria for scoring. Some of these have been continued in culture now for three years and have remained typically tumorous. All the tumors which have been isolated thus far have been of the fila-

mentous pseudocallus[1] type. Also, in those strains which were examined cytologically, the characteristic polyploidy was observed, as had been described earlier in the spontaneous tumors.[2] Thus, in every way these radiation-induced tumors seem comparable to those which occur spontaneously.

The effective dosage ranges differ somewhat with the condition of the object being irradiated. The growing prothalli are much more sensitive to X-radiations than are the dormant spores. Tumor production by irradiation of spores appears to increase sharply in the range of 10,000–20,000 r, while the lethal range has not been found, although doses up to 40,000 r have been given.[6] The optimal tumor-producing range for young prothalli seems to be in the range of 1,000–2,000 r of X-rays, doses in the range of 8,000 r being lethal.

The dose-effect relationship for the tumor response must at present be stated cautiously and only approximately, since the exact quantification of the phenomenon encounters several difficulties. Although the total dose of radiation can be expressed quite precisely, the expression of the response is less exact. An obvious way to score the cultures would be simply to count the numbers of tumors on some unit basis, e.g., per culture. However, the culture as a unit is meaningless unless the inocula are constant. In earlier experiments the inocula were rather variable, and in more recent ones constancy probably has been only approximated. Ideally, a known, relatively small number of spores, e.g., 1,000, would be uniformly distributed on the surface of an agar slant. The spores would be apart from one another and not too densely spaced, so that it would be feasible to make viability counts as well as counts of the total population. Many of these conditions have been realized in recent experiments, by using weighed amounts of spores suspended in measured amounts of water containing a trace of a wetting agent (digitonin), to prevent clumping, and using measured amounts of this suspension as inocula. However, as the ideal is approached experimentally, a well-known biological phenomenon is encountered, i.e., that often the size or density of the inoculum in relation to the quantity, or perhaps simply the surface, of the medium is a definite factor in the germination and growth of the inoculum.[5] Inocula which are sparse enough to meet the other conditions very often do not germinate. Most of these technical problems can, however, at least be minimized.

Another type of problem encountered in the quantification of the tumor response is that of positive recognition of tumors at an early stage. Tumorous nodules of 1 mm. or more in diameter can quite readily be recognized; however, smaller nodules are more difficult to identify, and, also, not all of them appear at the same time. Furthermore, some tumors are short-lived, while others appear to be potentially unlimited in their growth, with all degrees of intermediates. Therefore, the criteria for identification have of necessity been rather arbitrary in these many respects.

Current work is being directed in part toward earlier recognition of the tumors, ideally at the cellular level. However, although a tumor is quite distinctive as a group of cells, there seem to be no readily obvious distinguishing characteristics at the cellular level, at least not during the early stages. That is, almost every cell type that can be found in a tumor can probably also be found in what we consider to be nontumorous growth.

However, despite certain difficulties in the exact quantitative expression of the response, the fact that tumors can be produced at will and that they are being produced in vitro under aseptic and easily controlled conditions makes this a valuable technique. On the basis of present observations, it is rather tempting to suggest that this type of tumorization could be at least in part due to a mutational change, especially in light of the effectiveness of radiations on dormant spores. As the quantitative aspects of the process are refined, the natural and induced rates of occurrence can be stated more precisely, preferably in terms of numbers of spores. This should permit quantitative studies on various factors which in any way affect tumor formation and, it is hoped, may offer some insight into some of the more fundamental underlying mechanisms.

Summary.—Spontaneous tumorous abnormalities occur relatively rarely in aseptic in vitro cultures of the prothalli of *P. aquilinum* (the bracken fern). By treating either ungerminated spores or young prothalli with either X-rays or gamma radiation, tumors, which appear in every way comparable to the spontaneous ones, can be produced in quantity. With further refinement of the quantification of this technique, it promises to become a valuable tool for investigations into problems of tumorization and some of the factors involved.

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† Fellow in Cancer Research of the American Cancer Society.

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[6] Results of a current experiment involving high doses of gamma irradiation show that the lethal range for dry spores of *P. aquilinum* is in the vicinity of 100,000 r since there was some survival at that dose but none at 200,000 r.

*A NEW METHOD FOR THE MEASUREMENT OF SELF-DIFFUSION**

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In the search for a means of measuring the coefficients of self-diffusion of radioactive isotopes in aggregates of clay minerals, the procedure described here has been devised. The method is applicable to wet solids of almost any character and with minor modifications can be applied to solutions as well. It promises to be of quite general utility.

The material under investigation, in the form of a short rod of known length (or, if a semisolid is in question, it may be contained in a cylindrical cup of known depth), containing the radioactive isotope, is brought into the well of a scintillating crystal. The decrease in the total radioactivity is followed as the diffusion takes place, perhaps through a thin membrane, into a rapidly moving solution at zero activity. The advantages of the method are twofold. It supplies a nearly continuous record of the course of the diffusion, thereby providing much information and so increasing the accuracy of the experiment. (This continuous record also makes it possible to observe at once when anything goes wrong.) Furthermore, the nature of the experiment is such that conditions may be easily adjusted to give in a very few hours a determination even of a relatively low coefficient of diffusion. Comprehensive investigations thus become feasible.

The theory of the experiment can be given completely in so far as diffusion in systems at chemical equilibrium is concerned. In this case the problem is linear, and no mathematical difficulties arise. When the method is applied to systems not in chemical equilibrium, many complications appear, both in theory and in practice. These have not yet been seriously examined.

Although it is not always necessary to separate the diffusing system from the flowing fluid by a membrane (e.g., in the case of a relatively hard resinous ion-exchanger), the theory will be given assuming the presence of a membrane. The simpler situation will appear as a special case.

Consider a short rod impervious at its sides and at $x = 0$, capped at $x = a$ with a membrane b thick. At $x = a + b$ the membrane is in contact with a violently stirred solution at zero radioactivity. Let the concentration in the rod of the chemical species in question be q and that in the membrane q_m, and let θ and θ_m be the fractions of these atoms in rod and membrane, respectively, which are radioactive. If D and D_m are the respective coefficients of diffusion, the problem may be phrased as follows:

$$D\frac{\partial^2\theta}{\partial x^2} = \frac{\partial\theta}{\partial t}, \qquad 0 \leq x \leq a, \tag{1}$$

$$\frac{\partial\theta}{\partial x} = 0, \qquad x = 0, \tag{2}$$

$$\theta = \theta_m, \qquad x = a, \tag{3}$$

$$qD\frac{\partial\theta}{\partial x} = -q_m D_m \frac{\theta}{b}, \qquad x = a, \tag{4}$$

$$\theta = \theta_0, \qquad t = 0. \tag{5}$$

Equation (3) introduces the idea of local isotopic equilibrium at the interface of rod and membrane. Equation (4), which utilizes equation (3), expresses the equality of flux out of the rod into the membrane and introduces the assumption that the gradient of the concentration of the labeled isotope can be considered to be linear within the membrane. This concentration drops to zero at the interface between membrane and flowing solution.

The problem can be immediately solved by the method of the Laplace transform. To get the transform of the total activity Q in the rod, we have

$$L[Q] = \int_0^a L[\theta]\, dx, \tag{6}$$

in which we neglect, temporarily, the activity within the membrane itself. One finds for the ratio of total activity in the rod to its initial total activity

$$\frac{Q}{Q_0} = 1 - \frac{1}{\pi i}\frac{1}{a}\int_L \frac{e^{\lambda^2 Dt} d\lambda}{\lambda^2} \cdot \frac{e^{a\lambda} - e^{-a\lambda}}{(\lambda\xi + 1)e^{a\lambda} - (\lambda\xi - 1)e^{-a\lambda}}. \tag{7}$$

Here ξ is a parameter expressing the effective thickness of the membrane:

$$\xi = b\frac{Dq}{D_m q_m}. \tag{8}$$

The Bromwich integral in equation (7) may be evaluated in a variety of ways, giving results useful at various stages of the experiment. Only two of these need be considered; they overlap and, between them, give a procedure for computation covering all times. If the second term of the integrand is expanded in decreasing exponentials, the resulting series is convergent for $R(\lambda\xi) > 0$. The decreasing exponential makes for rapid convergence, and the integral of the first two terms gives a formula useful for short and intermediate times. This result has been expressed in terms of the Hh_n functions tabulated by the British Association:[1]

$$\begin{aligned}\frac{Q}{Q_0} = 1 &- \frac{2}{a\sqrt{\pi}}\sqrt{Dt}\left\{1 - 2Hh_1\left(a\sqrt{\frac{2}{Dt}}\right) - \right. \\ &\left. 2e^{Dt/\xi^2 + 2a/\xi} Hh_1\left(\sqrt{2}\left[\frac{\sqrt{Dt}}{\xi} + \frac{a}{\sqrt{Dt}}\right]\right) + \cdots\right\} \\ &+ \frac{\xi}{a}\left\{1 - \sqrt{\frac{2}{\pi}} e^{Dt/\xi^2} Hh_0\left(\frac{1}{\xi}\sqrt{2Dt}\right) - \frac{2^{3/2}}{\sqrt{\pi}} Hh_0\left(a\sqrt{\frac{2}{Dt}}\right)\right. \\ &\left. + \frac{2^{3/2}}{\sqrt{\pi}} e^{Dt/\xi^2 + 2a/\xi} Hh_0\left(\sqrt{2}\left[\frac{\sqrt{Dt}}{\xi} + \frac{a}{\sqrt{Dt}}\right]\right) + \cdots\right\}.\end{aligned} \tag{9}$$

At moderately short times entirely adequate accuracy is afforded by neglecting the corrections in the first brace and all but the first Hh_0 in the second.

Perhaps the most instructive and useful form for equation (7) is the result obtained through a development of the second term of the integrand in powers of ξ. The resulting series can easily be seen to be asymptotic in $1/\lambda$, in the Poincaré sense, and so can be integrated $d(1/\lambda)$, which is just what is required. This procedure gives a formula in which the effects of the membrane appear as corrections to the (exact) integral for the case of no membrane:

$$\begin{aligned}\frac{Q}{Q_0} = 1 &- \frac{2}{a\sqrt{\pi}}\sqrt{Dt}\left\{1 - 2\sum_{n=1}^{\infty}(-1)^{n+1} Hh_1\left(na\sqrt{\frac{2}{Dt}}\right)\right\} \\ &+ \frac{\xi}{a}\left\{1 - 4\sqrt{\frac{2}{\pi}}\sum_{n=1}^{\infty}(-1)^{n+1} nHh_0\left(na\sqrt{\frac{2}{Dt}}\right)\right\} \\ &- \frac{\xi^2}{a\sqrt{\pi Dt}}\left\{1 - 6e^{-a^2/Dt} + 18e^{-4a^2/Dt} - 38e^{-9a^2/Dt} + \cdots\right\}\end{aligned} \tag{10}$$

$$- \frac{8\xi^3}{\sqrt{\pi}(Dt)^{3/2}} e^{-a^2/Dt} \left\{1 - 8e^{-3a^2/Dt} + 33e^{-8a^2/Dt} - 96e^{-15a^2/Dt} + \ldots\right\}$$

$$+ \frac{\xi^4}{2a\sqrt{\pi}(Dt)^{3/2}} \left\{1 - 10\left(1 - \frac{2a^2}{Dt}\right) e^{-a^2/Dt} + \ldots\right\}$$

$$+ \frac{6\xi^5}{\sqrt{\pi}(Dt)^{5/2}} e^{-a^2/Dt} \left\{\left(3 - \frac{2a^2}{Dt}\right) + \ldots\right\}$$

$$+ \ldots.$$

This result is not valid at very short times, but in spite of its appearance it gives an easy way to compute the course of the diffusion after the initial period of the experiment is over.

It is evident from both equation (9) and equation (10) that, to a first approximation, after the initial stages, the course of Q/Q_0 is a straight line in $\sqrt{t}$ and that, when extended to $\sqrt{t} = 0$, this line passes through $Q/Q_0 = 1 + \xi/a$. This fact affords the means for handling the experimental data. In the absence of a membrane, a straight line is obtained which extends from $Q/Q_0 = 1$ to $Q/Q_0 \simeq 0.3$. The slope of this line gives $\sqrt{D}$ at once. If a membrane is in use, a straight line is drawn through intermediate points. By using approximate values of ξ and D, this intermediate region can be defined, with the aid of equations (9) and (10), in terms of the statistical accuracy of the radioactivity measurements. If desired, an iteration will give an improved value of D.

As an illustration of the above discussion, Figure 1 gives the result of an experiment on the self-diffusion of Na 22 in a mass of attapulgite clay wet with 0.1 N NaCl and confined in a small Dural cup of known depth by a membrane of unwaterproofed cellophane. The gamma activity was followed by a scintillation spectrometer of high resolution equipped with five scales of 10, using a thallium-activated sodium iodide crystal. Since an experiment cannot be started with the required straight-line gradient in the membrane, the first effect observed is a small sharp drop in activity due to the leaching of the membrane. The magnitude of this decrease could readily be measured in a separate experiment, but in the case here given it has merely been estimated. The computed values of Q/Q_0, given as a solid line in the figure, result from applying the factor $(1 + {}^1/_2(Q/Q_0)\alpha)/(1 + \alpha)$ to the results of the computation from equations (9) and (10). The initial and residual activity in the membrane are thereby largely taken into account. (Evidently this correction procedure can be easily refined.) The values of D and ξ used in the computations were obtained from the slope and intercept of the straight line indicated in the figure. The initial drop in activity was taken to be 0.005 in Q/Q_0, giving $\alpha = 0.0101$. The agreement between computation and experiment is quite satisfying, even in this case where no iterations have been employed.

Evidently the thinnest possible membrane is the most desirable. A similar experiment with attapulgite using a much thicker membrane of Visking gave reasonable but generally less satisfactory agreement with the calculations based on first estimates of D and ξ. Experiments with cylindrical slices from resinous ion-exchangers using no membrane give the expected simple straight lines.

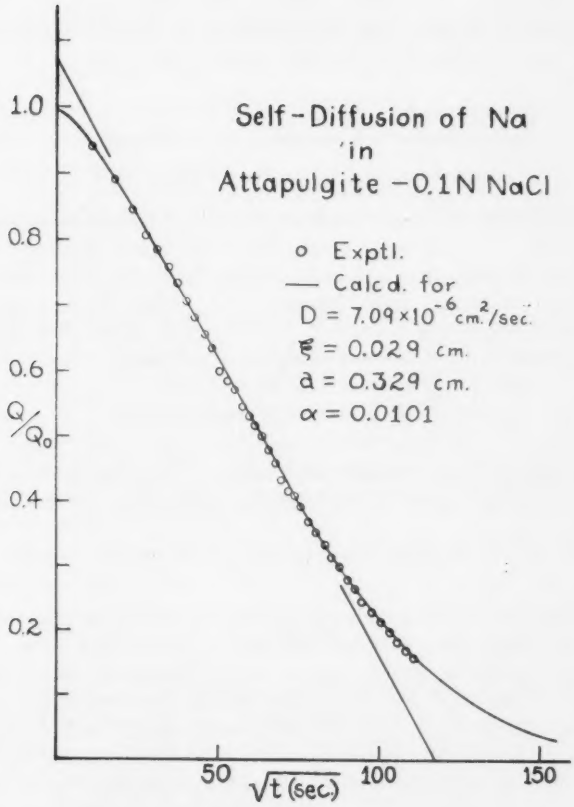

FIG. 1

The author wishes to express to the American Petroleum Institute and especially to its Clay Research Committee his best thanks for a generous grant which made possible the purchase of the equipment used in this work.

* Contribution No. 1405 from the Sterling Chemistry Laboratory, Yale University.

[1] British Association for the Advancement of Science, *Mathematical Tables*, **1**, (London, 1931), 60.

PARABOLIC SYSTEMS OF DIFFERENTIAL EQUATIONS WITH TIME-DEPENDENT COEFFICIENTS

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Let G_t for each real $t > 0$ be an open set in E^n, the Euclidean n-space with coordinates $x = (x_1, \ldots, x_n)$ $(n \geq 1)$, $A(t)$ and $B(t)$ two systems of r differential operators $(r \geq 1)$ of orders $2m$ and $2s$, respectively $(2s \leq 2m - 2)$, with coefficients defined on G_t and acting on r-vector functions on G_t. Let $A(t)$ be strongly elliptic and $B(t)$ formally self-adjoint for each t. In Section 1 we establish necessary and sufficient conditions on ζ for the existence of weak solutions of the equation

$$\frac{\partial}{\partial t} B(t)u + (-1)^m A(t)u = \zeta, \tag{1.1}$$

satisfying the null Dirichlet boundary conditions in G_t for almost all t. In Section 2 we outline a proof by energy methods of the regularity of the weak solutions of

$$\frac{\partial}{\partial t} u + (-1)^k A(t)u = \zeta. \tag{1.2}$$

Section 3 outlines the proof of the existence and uniqueness of the solution of the mixed initial-boundary-value problem for equation (1.2) with $k = m$. A detailed exposition with generalizations to general elliptic boundary conditions and the differentiability of solutions at the boundary will be presented in another publication. An existence proof for weak solutions using energy methods has been announced by J. L. Lions.[1] An extension of the Hille-Yosida theorem which covers the existence of weak solutions to mixed initial-boundary problems in some special cases has been given by T. Kato.[2] A differentiability proof for weak solutions of parabolic systems by the parametrix method under stronger differentiability hypotheses has been given by S. Mizohata.[3]

1. Let $G = \bigcup_{0<t<T} (\{t\} \times G_t)$, for some $T > 0$. Let G be a bounded open set of E^{n+1} with A and B of the form

$$\{A(t)u\}_i = \sum_{j=1}^{r} \sum_{|\alpha|,|\beta| \leq m} D_x^{\beta}(a_{\alpha\beta ij} D_x^{\alpha} u_j) \qquad (i = 1, \ldots, r), \tag{1.3}$$

$$\{B(t)u\}_i = \sum_{j=1}^{r} \sum_{|\alpha|,|\beta| \leq s} D_x^{\beta}(b_{\alpha\beta ij} D_x^{\alpha} u_j) \qquad (i = 1, \ldots, r), \tag{1.4}$$

with $a_{\alpha\beta ij}$ uniformly continuous, $b_{\alpha\beta ij}$ and $\partial b_{\alpha\beta ij}/\partial t$ uniformly bounded functions on every bounded subset of G; $b_{\alpha\beta ij} = b^*_{\beta\alpha ji}$. (For a multi-index $\alpha = (\alpha_1, \ldots, \alpha_k)$, $|\alpha| = k$, $D_x^{\alpha} = \partial^k/\partial x_{\alpha_1} \ldots \partial x_{\alpha_k}$. We assume that $A(t)$ is strongly elliptic for each t, i.e., there exists $q(t) > 0$, continuous for $t \geq 0$, such that for every real n-vector ξ and real r-vector η, $(\xi^{[\alpha]} = \xi_{\alpha_1} \ldots \xi_{\alpha_k})$, $x \in G_t$,

$$\sum_{i,j=1}^{r} \sum_{|\alpha|,|\beta|=m} a_{\alpha\beta ij}(x, t)\xi^{[\alpha]} \xi^{[\beta]} \eta_i \eta_j \geq q(t) \left(\sum_{i=1}^{r} \eta_i^2 \right) \left(\sum_{|\beta|=m} \xi^{[\beta]2} \right). \tag{1.5}$$

If ϕ lies in $C_c^{\infty, r}(G)$, the infinitely differentiable r-vector functions with compact support in G, $j \geq 0$, let $\|\phi\|_{j,(x)}^2 = \sum_{|\alpha| \leq j} (D_x^\alpha \phi, D_x^\alpha \phi)$, where $(\phi, \psi) = \sum_{j=1}^{r} \int_G \phi_j \psi_j^* \, dx \, dt$. Let $H_{m,(x)}(G)$ be the completion of $C_c^{\infty, r}(G)$ with respect to $\| \ \|_{m,(x)}$. The identification mapping of $C_c^{\infty, r}(G)$ with a subset of $L^{2,r}(G)$ may be extended by continuity to a continuous one-to-one mapping of $H_{m,(x)}(G)$ into $L^{2,r}(G)$. The subset of $L^{2,r}(G)$ with which $H_{m,(x)}(G)$ is identified consists of functions, each of which satisfies the null Dirichlet boundary conditions in the variational sense on the boundary of G_t for almost all t.

Let $a^1_{\alpha\beta ij} = {}^1/_2 \{a_{\alpha\beta ij} + a_{\beta\alpha ji}\}$, $a^2_{\alpha\beta ij} = {}^1/_2 \{a_{\alpha\beta ij} - a_{\beta\alpha ji}\}$, $b^1_{\alpha\beta ij} = \partial b_{\alpha\beta ij}/\partial t$. To the family of functions $a_{\alpha\beta ij}$ we assign the form on $C_c^{\infty, r}(G)$, $[\phi, \psi]_A = \sum_{i,j=1}^{r} \sum_{\alpha, \beta} (-1)^{|\beta|} (a_{\alpha\beta ij} D_x^\alpha \phi_j, D_x^\beta \psi_i)$. Similarly, we introduce the forms corresponding to the operators A_1, A_2, B, and B_1 with the families of functions $a^1_{\alpha\beta ij}$, $a^2_{\alpha\beta ij}$, $b_{\alpha\beta ij}$, $b^1_{\alpha\beta ij}$, respectively. Each of these forms may be extended by continuity to $H_{m,(x)}(G)$, while there exists $\rho > 0$, $k_0 \geq 0$, such that $(-1)^m [u, u]_{A_1} \geq \rho \|u\|^2_{m,(x)} - k_0(u, u)$ for $u \in H_{m,(x)}(G)$. If we introduce the inner product $[\phi, \psi] = (-1)^m [\phi, \psi]_{A_1} + k_0(\phi, \psi)$, the completion H of $C_c^{\infty, r}(G)$ with respect to this inner product is equivalent to $H_{m,(x)}(G)$.

If u is a regular solution of (1.1) and A and B have regular coefficients, integration by parts yields:

$$\left[u, \frac{\partial \psi}{\partial t}\right]_B + (-1)^m [u, \psi]_A = (\zeta, \psi), \qquad \psi \in C_c^{\infty, r}(G). \tag{1.6}$$

An element u of $H_{m,(x)}(G)$ is said to be a weak solution of equation (1.1) if equation (1.6) holds. We define $[\phi, \psi]_S = [\phi, \partial\psi/\partial t]_B - {}^1/_2 [\phi, \psi]_{B_1} + (-1)^M [\phi, \psi]_{A_2}$, $[\phi, \psi]_C = {}^1/_2 [\phi, \psi]_{B_1} - k_0(\phi, \psi)$. Since $|[\phi, \psi]_S| \leq c(\psi) \|\phi\|_H$, there exists an operator S in H with domain $C_c^{\infty, r}(G)$ for which $[\phi, \psi]_S = [\phi, S\psi]$, while we have $[\phi, S\psi] = -[S\phi, \psi]$ for $\phi, \psi \in C_c^{\infty, r}(G)$. Similarly, $[\phi, \psi]_C = [\phi, C\psi]$, while $(\zeta, \psi) = [w, \psi]$ for a unique $w \in H$ and all $\psi \in C_c^{\infty, r}(G)$. Since $\|C\phi\|_H \leq c_1 \|\phi\|_{m-1,(x)}$, C is a compact operator in H. Rewriting equation (1.7) as $[u, \psi] + [u, \psi]_S + [u, \psi]_C = (\zeta, \psi)$, we obtain $[u, \psi + S\psi + C\psi] = [w \cdot \psi]$, or $(I + S^* + C^*)u = w$.

LEMMA 1. *Suppose C compact, S densely defined with $(-S) \subset S^*$. Let N be the nullspace of $(I + \bar{S} + C)$, $\bar{S}$ the closure of S. Then N is finite dimensional, and w lies in the range of $(I + S^* + C^*)$ if and only if w is orthogonal to N.*

Proof: Since N is the orthogonal complement of the closure of the range of $(I + S^* + C^*)$, it suffices to prove the range closed, with its orthogonal complement finite dimensional. We note first that $(I + S^*)$ is onto. For let $T = iS$; T is symmetric, and, by a theorem of Neumark,[4] there exists a self-adjoint extension T^+ of T in a Hilbert space H^+ containing H as a closed subspace. If $\zeta \in H$, let P^+ be the projection of H^+ on H, $u^+ = (-i)(T^+ - iI)^{-1} \zeta$, $u = P^+ u^+$. Then $[u, (I + S)\psi] = [u^+, (-i)(T + iI)\phi] = i[(T^+ - iI)u^+, \phi] = [\zeta, \phi]$, i.e., $(I + S^*)u = \zeta$.

Since the operator $T = I + S^*$ is closed and onto and $C_1 = C^*$ is compact, the range of $T + C_1 = (I + S^* + C^*)$ is closed and has a finite-dimensional orthogonal complement. For, if H_1 is the null space of T, $H_2 = H \ominus H_1$, $D_2 = D(T) \cap H_2$, then $D(T) = H_1 - D_2$ and T maps D_2 one to one onto H. The inverse R of

$T|D_2$ is a closed and hence bounded operator from H to D_2. If $u \in D(T)$ ($u = u_1 + u_2$, $u_1 \in H_1$, $u_2 \in D_2$), then $(T + C_1)u = Tu_2 + C_1u_1 + C_1u_2$. Since the general element of D_2, $u_2 = Rv$, where v is the general element of H, $(T + C_1)u = \zeta$ if and only if $v + C_1Rv = \zeta - C_1u_1$. Since C_1R is compact, ζ will lie in the range of $T + C_1$ if and only if for some $u_1 \in H_1$, $[\zeta - C_1u_1, w] = 0$ for the finite-dimensional family F of solutions w of the equation $(I + R^*C_1^*)w = 0$, i.e., the projection of ζ into F should lie in the subspace $P_FC_1(H_1)$. *Q.E.D.*

THEOREM 1. *If B is positive, i.e., $[\phi, \phi]_B > c_0(\phi, \phi)$ for some $c_0 > 0$ and all $\phi \in C_c^{\infty,r}(G)$, then equation* (1.1) *will have weak solutions for every ζ. The same is true for negative B.*

Proof: If we introduce the new variable $v = ue^{\lambda t}$, A will be replaced by $(A - \lambda B)$. Then, for sufficiently large λ, positive λ, $I + C$ will have a positive self-adjoint part, and $N = \{0\}$.

2. Let u be a weak solution of equation (1.2): $u \in H_{m,(x)}(G)$,

$$-\left(u, \frac{\partial \phi}{\partial t}\right) + (-1)^k [u, \phi]_A = (\zeta, \phi), \phi \in C_c^{\infty,r}(G). \tag{2.1}$$

Let $j(x, t) \in C_c^{\infty}(E^{n+1})$, with the support of j contained in the unit ball around the origin, $j(x, t) \geq 0$, $\int_{E^{n+1}} j(x, t)\, dx\, dt = 1$. Put $j_\epsilon(x, t) = \epsilon^{-n} j(\epsilon^{-1}x, \epsilon^{-1}t)$, $(J_\epsilon u)_t(x, t) = \int_G j_\epsilon(x - x^1, t - t^1)u(x^1, t^1)\, dx^1\, dt^1$. Let $p \in C^{\infty}(G)$, Δ_x = the Laplace operator in E^n, $s \geq 0$. There exist constants $c_s > 0$, $k_s \geq 0$, such that

$$c_s\|pJ_\epsilon u\|^2_{m+s,(x)} - k_s\|pJ_\epsilon u\|^2_0 \leq \left| Re\left\{\left(pJ_\epsilon u, \frac{\partial}{\partial t}\, \Delta_x^s\, (pJ_\epsilon u)\right) + (-1)^k [pJ_\epsilon u, \Delta_x^s (pJ_\epsilon u)]_A\right\}\right|. \tag{2.2}$$

It follows from equation (2.1) by a calculation familiar in the theory of elliptic equations[5] that:

$$\left(pJ_\epsilon u, \frac{\partial}{\partial t}\, \Delta_x^s (pJu)\right) + (-1)^k [pJ_\epsilon u, \Delta_x^s\, (pJ_\epsilon u)]_A = (pJ_\epsilon u, \Delta_x^s (pJ_\epsilon \zeta)) + \sum_{|\beta| \leq s+m} (D_x^\beta (pJ_\epsilon u), C_x^\beta\, (p, \epsilon)\, (p_1 u)), \tag{2.3}$$

where $p_1 \in C_c^{\infty}(G)$ and equals 1 on the support of p, while, for a series of constants $k_\beta(p)$, independent of ϵ, the systems of differential operators C_x^β satisfy the inequality $\|C_x^\beta(p, \epsilon)\, (p_1 u)\|_0 \leq k_\beta \|p_1 u\|_{m+s-1,(x)}$, provided that $p_1 u$ has a finite $(m + s - 1)$ norm and the coefficients of A lie in $C^s(G)$. Combining these inequalities with relations (2.2) and (2.3) and applying induction on s, we obtain

THEOREM 2. *Suppose $D_x^\xi a_{\alpha\beta ij} \in C^0(G)$ for $|\xi| \leq s$, while $D_x^\xi \zeta$ (in the distribution sense) lies in $L^{2,r}(G_1)$ for $|\xi| \leq s - m$ and every compact G_1 in G. Then the distribution derivatives $D_x^\xi u$ will lie in all $L^{2,r}(G_1)$ for $|\xi| \leq m + s$ with $\|pu\|^2_{m+s,(x)} \leq c_s' \|p\zeta\|^2_{s-m} + k_s' \|p_1 u\|^2_0$. If $m + s > [n/2] + L$, then $u = u_1$ a.e. in G, where $u_1/G_t \in C^L(G_t)$ for almost all t.*

From Theorem 2 and equation (2.1), it follows that if $a_{\alpha\beta ij}$ all lie in C^{2m} and ζ has a finite $(m, (x))$-norm on every compact of G, then $\partial u/\partial t$ will have a finite $(m,$

(x))-norm on every compact of G. Putting $\phi = \partial^{2k}/\partial t^{2k}\psi$ in equation (2.1) and applying the argument of Theorem 2 inductively on k, we obtain

THEOREM 3. *Every weak solution u of equation* (1.2) *will lie in $C^{j,r}(G)$ for any j, provided that ζ and the coefficients of A are sufficiently often differentiable.*

3. Suppose that, for each t_0 with $\epsilon(t_0) > 0$, there exist n-functions $y_1 = Y_i(x_1, \ldots, x_n, t)$ $(i = 1, \ldots, n)$ of class C_m on $\bar{G}$ for $|t - t_0| < \epsilon(t_0)$, mapping G_t in the x space on G_0 in the y-space. Since the existence and uniqueness of a solution of equation (1.2) with $k = m$ with null Dirichlet data and prescribed initial values need only be established in a neighborhood of each value of t to be valid over the whole t-interval, while the mapping from the x to y co-ordinates preserves the general form of the problem, we may assume without loss of generality that $G = G_0 \times I$ and that $|a_{\alpha\beta ij}(x, t) - a_{\alpha_i\beta_i ij}(x, 0)| < h$, where h is a prescribed small positive constant. For A with coefficients independent of t, the existence and uniqueness of the solution u_0 of equation (2.1) with initial value zero may be deduced from the Hille-Yosida Theorem,[6] and the result that every weak solution of equation (2.1) has initial values. If $R_0(\zeta) = u_0$, R_0 is a closed and hence bounded mapping from $H_{-m,(x)}(G)$[7] into $H_{m,(x)}(G)$, while $(A(0) - A(t))R_0$ is a mapping of small norm on $H_{-m,(x)}(G)$. If we let $u = R_0 v$, the existence of a solution of equation (2.1) with null initial values is equivalent to the equation for v, $v + (A(0) - A)R_0 v = \zeta$. But since $(A(0) - A)R_0$ has small norm, this equation has one and only one solution. From this fact we obtain

THEOREM 4. *There exists one and only one solution u of equation* (2.1) *with prescribed initial value $\phi \in L^{2,r}(G_0)$.*

[1] J. L. Lions, *Compt. rend. acad. sci.* (Paris), **242**, 3028–3030, 1956.

[2] T. Kato, *J. Math. Soc. Japan*, **5**, 208–234, 1953.

[3] S. Mizohata, *Compt. rend. acad. sci.* (Paris), **241**, 1547–1549, 1955.

[4] Cf. N. I. Achiezer and I. M. Glasman, *Theorie der linearen Operatoren in Hilbert Raum* (Berlin, 1954), p. 270.

[5] Cf. F. E. Browder, *Ann. Math. Study No.* **33**, pp. 15–61, 1954.

[6] Cf. P. D. Lax and A. N. Milgram, *Ann. Math. Study No.* **33**, pp. 167–190, 1954.

[7] $H_{-m,(x)}(G)$ is the dual of $H_{m,(x)}(G)$ with respect to the inner product $(,)$.

ON ABSOLUTE CONVERGENCE OF FOURIER SERIES

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I. Let $\varphi(t)$ be an even and integrable (Lebesgue's sense) function, having 2π as its period. Let

$$\varphi(t) \sim \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos nt.$$

Write

$$\Phi(t) = \int_0^t \varphi(u)\, du.$$

In this note, we intend to establish the following:

THEOREM I.[1] *If Σa_n converges absolutely, then $\Phi/t \to A$ as $t \to +o$, where A is the sum of Σa_n.*

Proof: Without loss of truth, we may obviously assume that both a_0 and A are zero. Write

$$\frac{\Phi}{t} = \left(\sum_{n=1}^{m} + \sum_{m+1}^{\infty}\right) a_n \frac{\sin nt}{nt} = \Phi_1 + \Phi_2,$$

where the value of m will be given later. Since Σa_n converges absolutely, for a given $\delta > 0$, we can choose a positive integer m_0 such that

$$|\Phi_2| \leqq \sum_{m+1}^{\infty} |a_n| < \delta \qquad (m \geqq m_0).$$

Denote

$$S_n = \sum_{\mu=n}^{\infty} \frac{a_\mu}{\mu},$$

and write

$$\begin{aligned}
\Phi_1 &= \sum_{1}^{m} a_n \frac{\sin nt}{nt} = \frac{1}{t}(S_n - S_{n+1}) \sin nt \\
&= \frac{\sin t}{t} \sum_{1}^{m} S_n - \frac{\sin t}{t} \sum_{1}^{m} (1 - \cos (n-1)t) \\
&\quad - \frac{1 - \cos t}{t} \sum_{1}^{m} S_n \sin (n-1)t - \frac{1}{t} S_{m+1} \sin mt \\
&= \Phi_3 + \Phi_4 + \Phi_5 + \Phi_6.
\end{aligned}$$

Since the absolute convergence of Σa_n implies the convergence of the same series, for the given $\delta > 0$, there corresponds a positive integer n_0 such that

$$|a_n + a_{n+1} + \ldots + a\mu| < \delta$$

for all μ and n satisfying $n_0 \leqq n < \mu$. Thus Abel's transformation gives

$$|S_n| = \left| \sum_{\mu=n}^{\infty} \frac{a_n + a_{n+1} + \ldots + a_\mu}{\mu(\mu+1)} \right| < \frac{\delta}{n}$$

for $n \geqq n_0$. By a similar argument, we obtain

$$|S_n| < \frac{M}{n}$$

for all n, where M is the maximum value of $|a_n + a_{n+1} + \ldots + a_n|$ for all n and $\mu(>n)$. Write

$$\sum_{0}^{n} S_\mu = \sum_{0}^{n} a_\mu + (n+1) \sum_{n+1}^{\infty} \frac{a_\mu}{\mu} = \sum_{0}^{n} a_\mu + (n+1) S_{n+1}.$$

We see that ΣS_n converges to the same sum of Σa_n, since $S_n = o(n^{-1})$.

Let

$$m = \left[\frac{1}{t}\right].$$

By the above analysis for the sequence (S_n), we have

$$|\Phi_6| \leqq m|S_{m+1}| < m\frac{\delta}{m+1} < \delta$$

if $m \geqq n_0$. Also, since Σa_n converges to zero, for the given δ, we can choose a positive integer n_1 such that

$$|\Phi_3| \leqq |\sum_1^m S_n| < \delta$$

for $m \geqq n_1$. Furthermore,

$$\begin{aligned}|\Phi_4| &\leqq \frac{1}{2}t\sum_1^m n^2|S_n| \\ &= \frac{1}{2}t^2\left(\sum_1^{n_0-1} + \sum_{n_0}^m\right) n^2|S_n| \\ &< \frac{1}{2}t\cdot Mn_0^2 + \frac{1}{2}t^2\delta m^2 \\ &\leqq \frac{1}{2}t^2Mn_0^2 + \frac{1}{2}\delta.\end{aligned}$$

Finally,

$$|\Phi_5| \leqq \frac{t}{2}\sum_1^m |S_n| < Mt\log m \leqq Mt\log\frac{1}{t}.$$

We take t', satisfying

$$\left[\frac{1}{t'}\right] = m_1 = \max(n_0, n_1, m_0),$$

and write

$$\frac{\Phi}{t} = \left(\sum_1^{m_1} + \sum_{m_1+1}^{\infty}\right) a_n\frac{\sin nt}{nt} = \Phi_1 + \Phi_2.$$

Then we obtain

$$\left|\frac{\Phi(t)}{t}\right| \leqq |\Phi_1(t) + |\Phi_2(t)| < \frac{7}{2}\delta + Mt\log\frac{1}{t} + \frac{1}{2}t^2Mn_0^2$$

for $t \leqq t'$. For a given $\epsilon > 0$, we take $\delta = {}^1/_7\epsilon$. Then the values of m_0, n_0, and n_1 are simultaneously determined. Accordingly, t' and m_1 are designated. Last, for the fixed n_0, we choose $t''(<t')$ satisfying

$$Mt''\log\frac{1}{t''} + {}^1/_2\, t''^2Mn_0^2 < \frac{\epsilon}{2}.$$

Denote

$$\left[\frac{1}{t''}\right] = m_2,$$

and write

$$\frac{\Phi}{t} = \left(\sum_1^{m_2} + \sum_{m_2+1}^{\infty}\right) a_n \frac{\sin nt}{nt} = \Phi_1 + \Phi_2.$$

Then it follows immediately that

$$\left|\frac{\Phi(t)}{t}\right| \leqq |\Phi_1(t)| + |\Phi_2(t)|$$

$$\leqq |\Phi_1(t)| + |\Phi_3(t)| + |\Phi_4(t)| + |\Phi_5(t)| + |\Phi_6(t)|$$

$$< \frac{\epsilon}{2} \cdot 2 = \epsilon$$

for $t \leqq t''$. E.g., $\Phi(t) = o(t)$ as $t \to 0$. This proves the theorem.

2. Now, let $\psi(t)$ be an odd function. Suppose that

$$\Psi(t) \sim \sum_{n=1}^{\infty} b_n \sin nt.$$

Write

$$\Psi(t) = \int_0^t \psi(u)\, du.$$

By considering

$$\frac{\Psi(t)}{t} = \sum_1^{\infty} b_n \frac{1 - \cos nt}{nt} = \sum_1^{\infty} b_n \frac{\sin^2 nu}{nu} \qquad \left(u = \frac{t}{2}\right)$$

and the same arguments as in the proof of Theorem I, we establish

THEOREM 2. *If Σb_n converges absolutely, then $\Psi(t)/t \to B$ as $t \to +\ddot{o}$, where B is the sum of Σb_n.*

[1] This theorem can be considered as the parallel theorem to Hardy-Littlewood's. Cf. G. H. Hardy and J. E. Littlewood, "On Young's Convergence Criterion for Fourier Series," *Proc. London Math. Soc.*, Ser. 2, **28**, 305–306, 1928.

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*A SEQUENTIAL DECISION PROBLEM WITH A FINITE MEMORY**

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1. *Summary.*—We consider the problem of successively choosing one of two ways of action, each of which may lead to success or failure, in such a way as to maximize the long-run proportion of successes obtained, the choice each time being based on the results of a fixed number of the previous trials.

2. *Introduction.*—An experimenter has two coins, coin 1 and coin 2, with respective probabilities of coming up heads equal to $p_1 = 1 - q_1$ and $p_2 = 1 - q_2$, the values of which are unknown to him. He wishes to carry out an infinite sequence of tosses, at each toss using either coin 1 or coin 2, in such a way as to maximize the long-run proportion of heads obtained. His problem is to find a rule for deciding at each stage, on the basis of the results of the previous tosses, whether to use coin 1 or coin 2 for the next toss.

If he knew at the outset which coin had the larger p-value, he would of course use it exclusively, irrespective of the results of the tosses, and by doing so would know that with probability 1

$$\lim_{n \to \infty} \frac{\text{number of heads in first } n \text{ tosses}}{n} = \max(p_1, p_2). \tag{1}$$

We have shown elsewhere[1] that even without a knowledge of the p-values there exist rules for carrying out the sequence of tosses such that equation (1) holds for all p_1, p_2. However, any rule with this property must clearly be such that the decision as to which coin to use for the nth toss depends on the results of all the first $n - 1$ tosses; in other words, the rule requires a "memory" of unlimited length. We shall be interested here in seeing what can be done with rules requiring only a *finite* memory: a rule will be said to be of type r if the decision as to which coin to use for the nth toss depends only on the results of tosses $n - r, n - r + 1, \ldots, n - 1$. (By the "result" of a toss we mean both which coin was used and which face came up.) We shall exhibt a rule R_r, of type r, for which with probability 1

$$\lim_{n \to \infty} \frac{\text{number of heads in first } n \text{ tosses}}{n} = \frac{p_1 q_2^r + p_2 q_1^r}{q_1^r + q_2^r}. \tag{2}$$

Note that as $r \to \infty$ the right-hand side of equation (2) steadily increases and tends to the right-hand side of equation (1) as limit.

It would be interesting to know whether our rule R_r is "best" in the sense that the right-hand side of equation (2) is at least as great, for all p_1, p_2, as the corresponding function for any other rule of type r for which the left-hand side of equation (2) exists and is a symmetric function of p_1, p_2. We do not know whether this is true.

3. *Definition of the Rule R_r and Proof of the Theorem.*—We recall[2] that if a single coin, with probability $q = 1 - p$ of obtaining tails on each toss, is tossed repeatedly until the first run of r consecutive tails occurs, then the expected number of tosses required is

$$\frac{1 - q^r}{pq^r}.$$

Now suppose the coin is tossed repeatedly with the following stopping rule: stop if the first toss is tails, otherwise continue tossing until the first run of r consecutive tails occurs. Then the expected number of tosses required will be

$$q \cdot 1 + p \cdot \left(1 + \frac{1 - q^r}{pq^r}\right) = \frac{1}{q^r}. \tag{3}$$

We now prove the following:

THEOREM. *Define the rule R_r as follows: start tossing with coin* 1. *Stop if the first toss is tails, otherwise continue tossing until the first run of r successive tails occurs and then stop. This defines the first block of tosses with coin* 1. *Now start tossing with coin* 2 *and apply the same rule, obtaining the first block of tosses with coin* 2. *Then start again with coin* 1 *and apply the same rule, obtaining the second block of tosses with coin* 1, *and so on indefinitely, thus generating an infinite sequence of tosses consisting of alternate blocks of tosses with coins* 1 *and* 2.

With rule R_r so defined, we assert that equatian (2) *hold with probability* 1.

Proof: Let $x_i(y_i)$ denote the length of the ith block of tosses with coin 1 (2), $i = 1, 2, \ldots$. The process of tossing generates with probability 1 an infinite sequence of independent random variables

$$x_1, y_1, x_2, y_2, \ldots, x_n, y_n, \ldots \tag{4}$$

The proportion of times that coin 1 is used during the first $2n$ *blocks* of tosses is then

$$\frac{x_1 + \ldots + x_n}{x_1 + \ldots + x_n + y_1 + \ldots + y_n} = \frac{(x_1 + \ldots + x_n)/n}{(x_1 + \ldots + x_n)/n + (y_1 + \ldots + y_n)/n}, \tag{5}$$

and, by the strong law of large numbers and equation (3), this tends with probability 1 to the limit

$$\frac{1/{q_1}^r}{1/{q_1}^r + 1/{q_2}^r} = \frac{{q_2}^r}{{q_1}^r + {q_2}^r}. \tag{6}$$

Now let u_n denote the proportion of heads obtained in the first $2n$ blocks of tosses; then

u_n = (proportion of times coin 1 is used in first $2n$ blocks of tosses)$\cdot$(proportion of times heads occurs among these tosses with coin 1) + a similar product for coin 2, (7)

so that with probability 1

$$\lim_{n \to \infty} u_n = \frac{{q_2}^r}{{q_1}^r + {q_2}^r} \cdot p_1 + \frac{{q_1}^r}{{q_1}^r + {q_2}^r} \cdot p_2 = \frac{p_1 {q_2}^r + p_2 {q_1}^r}{{q_1}^r + {q_2}^r}. \tag{8}$$

Now let n be any positive integer, and define the random integer $N = N(n)$ so that

$$x_1 + y_1 + \ldots + x_N + y_N \leq n < x_1 + y_1 + \ldots + x_{N+1} + y_{N+1}. \tag{9}$$

Denoting by w_n the number of heads obtained among the first n tosses, we have

number of heads in first $2N$ blocks $\leq w_n \leq$ number of heads in first $2N +$ 2 blocks, (10)

and from (9) and (10) it follows that

$$\frac{u_N(x_1 + y_1 + \ldots + x_N + y_N)}{x_1 + y_1 + \ldots + x_{N+1} + y_{N+1}} \leq \frac{w_n}{n} \leq$$

$$u_{N+1} \frac{(x_1 + y_1 + \ldots + x_{N+1} + y_{N+1})}{x_1 + y_1 + \ldots + x_N + y_N}. \tag{11}$$

But since $(x_{N+1} + y_{N+1})/N$ tends to 0 with probability 1 as n, and therefore N, becomes infinite, it follows that with probability 1

$$\lim_{n\to\infty} \frac{x_1 + y_1 + \ldots + x_{N+1} + y_{N+1}}{x_1 + y_1 + \ldots + x_N + y_N} = 1,$$

and hence with probability 1, from relations (8) and (11),

$$\lim_{n\to\infty} w_n/n = \lim_{N\to\infty} u_N = (p_1q_2^r + p_2q_1^r)/(q_1^r + q_2^r), \tag{12}$$

which was to be proved.

The author is indebted to John W. Tukey for a helpful remark which simplified the preceding proof.

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[1] H. Robbins, "Some Aspects of the Sequential Design of Experiments," *Bull. Am. Math. Soc.*, **58**, 529–532, 1952.

[2] W. Feller, *An Introduction to Probability Theory and Its Applications*, **1**, (New York, 1950), 266.

SLIP SURFACES IN PLASTIC SOLIDS

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1. *Introduction.*—In a previous communication[1] we considered the problem of the determination of the inclination of slip bands, conceived as strips of plastic material in an elastically deformed solid at the yield point, and on the basis of this conception we deduced an expression for the inclination of the bands which appear in the usual tension test on flat rectangular bars. The present paper[2] differs from the former in that (1) we deal with the slip surface *as such* and (2) we consider the entire solid, in which the slip surface is immersed, to be in the plastic state. This latter viewpoint is perhaps the one most commonly adopted.

It is assumed that the stress field is *plane*, by which it is meant that a system of rectangular co-ordinates x_1, x_2, x_3 can be chosen such that

$$\sigma_{\alpha 3} = 0 \quad (\alpha = 1, 2, 3); \qquad \sigma_{\alpha\beta} = \phi_{\alpha\beta}(x_1, x_2, t) \quad (\alpha, \beta = 1, 2), \tag{1}$$

where the σ's are the components of the stress tensor and the ϕ's are at most functions of the two co-ordinates x_1, x_2 and the time t. To the conditions (1) we must add the basic dynamical equations relating the discontinuities in the above components σ, the velocity components v_α, and the density ρ across a *shock surface*, namely,

$$\rho_1(v_{1n} - G) = \rho_2(v_{2n} - G), \tag{2}$$

$$[\sigma_{\alpha\beta}]\nu_\beta = \rho_1(v_{1n} - G)[\nu_\alpha], \tag{3}$$

in which the bracket [] denotes the jump or discontinuity in the quantity inclosed, while the subscripts 1 and 2 are used to indicate an evaluation on the sides 1 and 2 of the surface. Specifically, the quantities v_{1n} and v_{2n} are the normal velocity components along the unit normal ν, directed from side 1 to side 2, and G denotes the velocity of the surface in this normal direction; these velocities are relative to the co-ordinate system.

The condition of *slip* is expressed by the requirement that the vector with components $[v_\alpha]$ is (1) nonzero and (2) tangent to the above shock surface, i.e., $[v_\alpha]\nu_\alpha = 0$. Under this condition, $v_{1n} = v_{2n}$, and hence the normal velocity, which may now be denoted simply by v_n, is continuous across the slip surface. The *speed of propagation* of this surface is defined as the velocity difference $G - v_n$. In the following discussion we shall limit our attention to slip surfaces Σ whose speed of propagation is equal to zero. For such a surface equation (2) is satisfied identically, and equations (3) reduce to

$$[\sigma_{\alpha\beta}]\nu_\beta = 0. \tag{4}$$

The general formula (14) for the inclination of the above slip surfaces is derived solely from the postulate of plane stress (1), the quadratic yield condition (10), and certain assumptions on discontinuities in the stress. By neglecting the term involving $[\sigma_{12}]$ in this formula, we can deduce the value of the inclination, previously obtained in the literature,[3] for the slip surfaces in simple tension (see Sec. 5). A more refined analysis of the simple tension problem which does not necessitate the omission of the term in $[\sigma_{12}]$ may possibly result in better agreement between the observed and calculated inclinations of these slip surfaces.

2. *Discontinuities in the Stress.*—In this section we shall make two assumptions regarding the discontinuities $[\sigma_{\alpha\beta}]$ and the discontinuities $[\sigma_{\alpha\beta,\gamma}]$ in the derivatives of the components of the stress tensor. The first of these is the following:

ASSUMPTION A. *Not all the discontinuities* $[\sigma_{\alpha\beta}]$ *vanish at any point of the slip surface* Σ.

It will now be convenient to denote the time t by x_4 and hence to represent the combination of the space and time variables by writing x_i with $i = 1, 2, 3, 4$. Then, if $x_1 + \Delta x_1$, $x_2 + \Delta x_2$ and $x_3 + \Delta x_3$ are the co-ordinates of a point P' on the slip surface at a time $t + \Delta t$, we can readily deduce the relations

$$[\sigma'_{\alpha\beta}] = [\sigma_{\alpha\beta}] + [\sigma_{\alpha\beta,i}]\Delta x_i; \qquad \nu'_\beta = \nu_\beta + \nu_{\beta,i}\Delta x_i, \tag{5}$$

where the quantities $[\sigma'_{\alpha\beta}]$ and ν'_β are taken with reference to the point P' at time $t + \Delta t$ and the repeated index i is summed over the values 1, 2, 3, 4. In the second set of relations (5) we consider the quantities ν_β to be given by

$$\nu_\beta = \frac{F_{,\beta}}{\sqrt{F_{,\alpha}F_{,\alpha}}}, \tag{6}$$

where the surface Σ is defined by the equation $F(x, t) = 0$. Hence the derivatives $\nu_{\beta,i}$ which appear in relations (5) can be determined by differentiation of equation (6).

Multiplying corresponding members of the two sets of relations (5) and taking account of conditions of the form (4), we find

$$([\sigma_{\alpha\beta,i}]\nu_\beta + [\sigma_{\alpha\beta}]\nu_{\beta,i})\Delta x_i = 0, \tag{7}$$

for $F_{,i}\Delta x_i = 0$. From equation (7) we can now deduce relations of the form

$$[\sigma_{\alpha\beta,\gamma}]\nu_\beta + [\sigma_{\alpha\beta}]\nu_{\beta,\gamma} = \xi_\alpha\nu_\gamma, \tag{8}$$

$$\left[\frac{\partial\sigma_{\alpha\beta}}{\partial t}\right]\nu_\beta + [\sigma_{\alpha\beta}]\frac{\partial\nu_\beta}{\partial t} = -G\xi_\alpha, \tag{9}$$

in which the ξ_α are the components of a vector ξ defined over the slip surface Σ and the indices α, β, γ have values 1, 2, 3 only.

ASSUMPTION B. *The vector ξ is* (1) *nonzero and* (2) *not tangent to the slip surface Σ at any of its points.*

As a consequence of this assumption, the quantity $\xi_\alpha\nu_\alpha$ does not vanish on Σ. One must obviously expect this condition to be satisfied, in general, for any slip surface Σ over which discontinuities in the components $\sigma_{\alpha\beta}$ and their derivatives are assumed to exist.

Now multiply both members of equation (8) by ν_α and sum on the repeated index α. Then, using equation (4), we have

$$[\sigma_{\alpha\beta,\gamma}]\nu_\alpha\nu_\beta = \xi_\alpha\nu_\alpha\nu_\gamma.$$

Hence, putting $\gamma = 3$ and taking account of the second relation (1), it follows that $\xi_\alpha\nu_\alpha\nu_3 = 0$. Hence $\nu_3 = 0$ from Assumption B above. *In other words, the slip surfaces Σ are perpendicular to the x_1, x_2 co-ordinate plane.*

3. *Quadratic Yield Condition.*—We shall postulate the quadratic yield condition, which can be expressed by writing

$$\sigma^*_{\alpha\beta}\sigma^*_{\alpha\beta} = K, \tag{10}$$

where K is a positive material constant and the quantities $\sigma^*_{\alpha\beta}$ are the components of the so-called stress deviator. Putting $[\sigma_{\alpha\beta}] = \sigma_{\alpha\beta} - \bar{\sigma}_{\alpha\beta}$, so that the $\sigma_{\alpha\beta}$ and $\bar{\sigma}_{\alpha\beta}$ are the components of the stress tensor on opposite sides[4] of the slip surface Σ, and using the fact that the quantities $\bar{\sigma}^*_{\alpha\beta}$, derived from the $\bar{\sigma}_{\alpha\beta}$, also satisfy the yield condition (10), we can easily deduce the following relation:

$$[\sigma_{\alpha\beta}][\sigma_{\alpha\beta}] - {}^1/_3[\sigma_{ii}]^2 - 2\sigma^*_{\alpha\beta}[\sigma_{\alpha\beta}] = 0. \tag{11}$$

4. *Determination of the Slip Surfaces.*—If we take $\alpha = 3$ in relation (4), the equation is satisfied identically on account of the first relation (1). Hence there are at most two independent conditions in the set (4), and these can be expanded to give

$$[\sigma_{11}]\nu_1 + [\sigma_{12}]\nu_2 = 0; \qquad [\sigma_{21}]\nu_1 + [\sigma_{22}]\nu_2 = 0. \tag{12}$$

Denoting by θ the inclination of the slip surface Σ, we can put $\nu_1 = -\sin\theta$; $\nu_2 = \cos\theta$. The component ν_3 of the unit normal vector to Σ vanishes by the result in Section 2. Hence equations (12) become[5]

$$[\sigma_{11}] = [\sigma_{12}]\operatorname{ctn}\theta; \qquad [\sigma_{22}] = [\sigma_{12}]\tan\theta. \tag{13}$$

It now follows that $[\sigma_{12}] \neq 0$; otherwise all discontinuities $[\sigma_{\alpha\beta}]$ would vanish from equations (13) and the first set of equations (1), and this would contradict Assumption A.

Expanding equation (11) and making the substitution (13), the resulting equation can be written in the form

$$3(\sigma_1^* - \sigma_2^*) \sin 4\theta - 6\sigma_3^* \sin 2\theta - 8[\sigma_{12}] = 0, \tag{14}$$

relative to a system of canonical co-ordinates[6] for the unbarred components $\sigma_{\alpha\beta}$; the quantities σ_1^*, σ_2^*, and σ_3^* in this equation are the principal values of the stress deviator σ^*.

The following result has now been proved. *Under the assumption of plane stress* (1) *and the quadratic yield condition* (10), *a slip surface* Σ *having speed of propagation equal to zero and for which Assumptions A and B in Section* 2 *are satisfied will* (*a*) *be perpendicular to the* x_1, x_2 *co-ordinate plane and* (*b*) *have an inclination* θ *satisfying equation* (14) *at each of its points.* It may be emphasized that this result is independent of any particular system of stress-strain relations governing the plastic flow.

If we assume that the last term in equation (14) can be neglected in comparison with the other terms in this equation, we arrive at the formula

$$\cos 2\theta = \frac{\sigma_3^*}{\sigma_1^* - \sigma_2^*}, \tag{15}$$

which yields an immediate determination of the inclination θ when the components of the stress tensor, e.g., the components $\sigma_{\alpha\beta}$, are known on one side of the slip surface from the conditions of the problem.

The inclination θ can at most be a function of the two co-ordinates x_1, x_2 and the time t, since $v_3 = 0$. After this function has been found, e.g., by application of equation (14) or equation (15), the explicit equation for the slip surface Σ can be determined by an integration.

5. *Slip Surfaces in Simple Tension.*—If the stress tensor results from a simple tension in the direction of the x_2 axis, the principal directions determined by this tensor can be identified with the fixed co-ordinate directions, and we shall have

$$\sigma_2^* = \frac{2W}{3}; \qquad \sigma_1^* = \sigma_3^* = -\frac{W}{3}; \qquad K = \frac{2W^2}{3},$$

where W is the yield stress. Then equation (15) gives $\cos 2\theta = {}^1/_3$, and hence the slip surfaces will be planes whose intersection with the plane $x_3 = \text{const.}$ will make angles θ of numerical value 35°16′ with the x_1 axis. The discrepancy between this inclination and the observed inclination, which is approximately 30°, may arise, in part at least, from the fact that in determining the value 35°16′ we have neglected the term in $[\sigma_{12}]$ in equation (14).

[1] "On the Inclination of Plastic Slip Bands in Flat Bars in Tension Tests," these PROCEEDINGS, **39**, 257–265, 1953; see also "The Effect of Compressibility on the Inclination of Plastic Slip Bands in Flat Bars," *ibid.*, pp. 266–273.

[2] Prepared for the Applied Mathematics Branch, Mechanics Division, Naval Research Laboratory, Washington, D.C.

[3] See A. Nadai, *The Theory of Flow and Fracture of Solids*, (New York: McGraw-Hill Book Co., 1950), pp. 320–327.

[4] The components $\sigma_{\alpha\beta}$ are associated with side 2, and the $\bar{\sigma}_{\alpha\beta}$ with side 1 of the slip surface Σ, the unit normal vector ν being directed from side 1 to side 2 as stated in Sec. 1.

[5] We omit from consideration the possibility that $\theta = 0$ or $\pi/2$, since we are not aware that such inclinations are realized in practice.

[6] A discussion of canonical co-ordinates with special reference to the plane stress problem has been given by T. Y. Thomas, "On the Propagation of Weak Discontinuities in Perfectly Plastic Solids," *J. Math. and Mech.*, Vol. 6, 1957 (to appear).

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*NOTE ON DEGREE OF APPROXIMATION TO ANALYTIC FUNCTIONS BY RATIONAL FUNCTIONS WITH PREASSIGNED POLES**

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The purpose of this note is to establish several results, especially Theorem 1 below, relative to degree of approximation to analytic functions and their boundary values, the latter assumed to satisfy certain continuity conditions (Lipschitz conditions, etc.). The present results are closely related to, but more general than, results due to Sewell[1] and Elliott;[2] the results have application to degree of approximation by bounded analytic functions.

THEOREM 1. *Let E be a bounded open point set of the z-plane whose boundary J consists of a finite number of mutually disjoint analytic Jordan curves J_j, $J = \sum J_j$. Let $f(z)$ be analytic on E, continuous on $E + J$, and of class $L(p, \alpha)$ on J, $0 < \alpha < 1$. In the extended plane, let the set complementary to $E + J$ consist of the mutually disjoint regions $E_1, E_2, \ldots, E_\nu$, and for each k and n let points*

$$\alpha_{n1}^{(k)}, \alpha_{n2}^{(k)}, \ldots, \alpha_{nm_{nk}}^{(k)} \tag{1}$$

in E_k be given; $k = 1, 2, \ldots, \nu$; $n = \nu, \nu + 1, \nu + 2, \ldots$; $\sum_{k=1}^{\nu} m_{nk} = n$. Suppose that the points (1) have no limit points on J and that the numbers n/m_{nk} are bounded for all k and n. Then there exist rational functions $R_n(z)$ of respective degrees n whose poles lie in the points (1) such that we have

$$|f(z) - R_n(z)| \leqq \frac{A}{n^{p+\alpha}}, \qquad z \text{ on } E + J. \tag{2}$$

The class L (p, α) on J is the class of functions each having a one-dimensional pth derivative on J which satisfies there a Lipschitz condition of order α. A rational function of degree n whose poles lie in the points (1) is a function of the form

$$\frac{b_0 z^n + b_1 z^{n-1} + \ldots + b_n}{(z - \alpha_{n1}^{(1)}) \ldots (z - \alpha_{nm_{n\nu}}^{(\nu)})},$$

except that factors corresponding to possible infinite points among (1) are to be omitted. Here and below, the numbers A, with or without subscript, are constants independent of n and z which, however, may change from one formula to another.

For z in E we have

$$f(z) \equiv \sum_j \frac{1}{2\pi i} \int_{J_j} \frac{f(t)dt}{t - z}, \tag{3}$$

where the integrals are taken over all J, in the positive sense with respect to the regions composing E. Although the function

$$\frac{1}{2\pi i}\int_{J_j} \frac{f(t)\,dt}{t - z} \tag{4}$$

is defined and analytic at all points of the plane (including by continuity the point at infinity) except on J_j, it is not defined on J_j; we shall consider it to be defined on J_j by relation (3) as continuous in the closure of that region bounded by J_j which contains a subregion of E adjacent to J_j; we allow z in E to approach J_j, and in relation (3) use the limit on J_j of each of the other terms of (3); then the function (4) is of class $L(p, \alpha)$ on J_j and is analytic throughout that one of the two regions bounded by J_j which contains a subregion of E adjacent to J_j, continuous in the corresponding closed region.

Each curve J_j belongs to the boundary of precisely one region E_k. For every k, $1 \leqq k \leqq \nu$, we define the function

$$F_k(z) \equiv \sum_j \frac{1}{2\pi i}\int_{J_j} \frac{f(t)\,dt}{t - z}, \tag{5}$$

where the integrals are extended over the complete boundary of E_k, over each J_j of this boundary in the same sense as in relation (3); this function $F_k(z)$ is analytic in the interior points of the complement $C(E_k)$ of E_k (the complement contains $E + J$), continuous on $C(E_k)$, and by our interpretation of expression (4) is of class $L(p, \alpha)$ on the boundary of $C(E_k)$. By the theorem of Sewell and Elliott,[2] there exists a rational function $R_{m_{nk}}{}^{(k)}(z)$ of degree m_{nk} whose poles lie in the points (1), such that we have

$$\left|F_k(z) - R_{m_{nk}}{}^{(k)}(z)\right| \leqq \frac{A_k}{m_{nk}{}^{p+\alpha}}, \qquad z \text{ on } C(E_k).$$

If we assume $n/m_{nk} \leqq A_0$ for all n and k, we have $A_k/m_{nk}{}^{p+\alpha} \leqq A_k A_0{}^{p+\alpha}/n^{p+\alpha}$; hence the function $R_n(z) \equiv \sum_{k=1}^{\nu} R_{m_{nk}}{}^{(k)}(z)$ is a rational function of degree n whose poles lie in the points (1), which satisfies relation (2). Theorem 1 is established. The special case $\nu = 1$ of Theorem 1 and Corollary 1 is due to Sewell and Elliott (*op. cit.*).

Two corollaries are of interest.

COROLLARY 1. *If the hypothesis of Theorem* 1 *is modified by replacing the assumption that* $f(z)$ *is of class* $L(p, \alpha)$ *on* J *by the assumption that* $f^{(p)}(z)$ *exists and has the modulus of continuity* $\omega(s)$ *on* J, *where* s *is arc length on* J, *then there exist rational functions* $R_n(z)$ *of respective degrees* n *whose poles lie in the points* (1) *such that we have*

$$\left|f(z) - R_n(z)\right| \leqq \frac{A\omega(1/n)}{n^p}, \qquad z \text{ on } E + J. \tag{6}$$

The proof of Theorem 1 requires only minor changes to apply to Corollary 1. As before, for each k we obtain[2] a rational function of degree m_{nk} whose poles lie in the points (1) which approximates on $C(E_k)$ to the function $F_k(z)$, with an error now not greater than $A_k\omega(1/m_{nk})/m_{nk}{}^p$. Thanks to the relation[3]

$$\omega\left(\frac{1}{m_{nk}}\right) \leqq \left(\frac{A'n}{m_{nk}} + 1\right)\omega\left(\frac{1}{n}\right),$$

we obtain relation (6).

A function $f(z)$ defined on J is *of class Z_p on J*, provided that $f^{(p)}(z) \equiv f_1(s)$ exists and is continuous there, with

$$|f_1(s + h) + f_1(s - h) - 2f_1(s)| \leqq A_0|h|, \qquad z \text{ on } J,$$

in terms of arc length s; the corresponding class for trigonometric approximation was studied by Zygmund. The proof of Corollary 1 for the case $\nu = 1$ given by Elliott[2] carries over to the class Z_p, and, by the method of proof of Theorem 1, we have

COROLLARY 2. *If the hypothesis of Theorem 1 is modified so that $f(z)$ is of class Z_p on J instead of class $L(p, \alpha)$, there exist rational functions $R_n(z)$ of the prescribed type such that*

$$|f(z) - R_n(z)| \leqq \frac{A}{n^{p+1}}, \qquad z \text{ on } E + J. \tag{7}$$

Theorem 1 admits an extension:

THEOREM 2. *Let E be a bounded closed set whose boundary J consists of a finite number of mutually disjoint analytic Jordan curves J_j, $J = \sum J_j$. Let $f(z)$ be analytic in the interior points of E, continuous on E, and of class $L(p, \alpha)$ on J, $0 < \alpha < 1$. In the extended plane, let the set complementary to E consist of the mutually disjoint regions $E_1, E_2, \ldots, E_\nu$, and for each k and n let points (1) be given in E_k. Suppose that the points (1) have no limit points on J and that the numbers n/m_{nk} are bounded for all k and n. Then there exist rational functions $R_n(z)$ of respective degrees n whose poles lie in the points (1) such that relation (2) is valid.*

If the curve J_j is not adjacent to an open set in E, the integral (4) taken in the positive senses with reference to the interior and exterior of J_j, respectively, defines two functions analytic in those respective regions. These two functions (*components* of $f(z)$) approach[4] boundary values on J_j of class $L(p, \alpha)$ there, whose sum on J_j is $f(z)$. For z on E the second member of relation (3) is considered to contain two distinct integrals over each J_j not adjacent to an open set in E; these two integrals are in opposite directions, and their sum is $f(z)$ on this J_j, zero elsewhere on E. In relation (5) we interpret the (single) integral (4) over a curve J_j which belongs to the boundary of E_k, for points z on that J_j or separated by that J_j from E_k, as that one component of $f(z)$ defined by expression (4) in that region of the plane bounded by J_j which contains no points of E_k, including the continuous boundary values of that component on J_j itself; for points z neither on that J_j nor separated by that J_j from E_k the integral (4) in relation (5) is interpreted as the ordinary integral, in the positive sense with respect to the region (bounded by J_j) separated by J_j from E_k. With this additional interpretation, the proof of Theorem 1 is valid and establishes Theorem 2.

Corollary 2 to Theorem 1 has a precise analogue here:[4]

COROLLARY 1. *If the hypothesis of Theorem 2 is modified so that $f(z)$ is of class Z_p on J instead of class $L(p, \alpha)$, there exist rational functions $R_n(z)$ of the prescribed type such that relation (7) is valid.*

We mention in conclusion some results in the converse direction; it is clearly sufficient to consider approximation by rational functions on a single Jordan curve E. Converses of Theorem 1, its Corollary 2, Theorem 2, and its Corollary 1 have then been established;[5] in each case the appropriate continuity properties of $f(z)$ on J are consequences of such an inequality as relation (2) or relation (7). A converse of Corollary 1 to Theorem 1 requires[6] supplementary conditions on $\omega(s)$ or on degree of convergence.

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[1] W. E. Sewell, *Rev. cienc.*, **41**, 435–451, 1939.

[2] H. M. Elliott, *Proc. Am. Math. Soc.*, **4**, 161–167, 1953.

[3] H. M. Elliott, *Trans. Am. Math. Soc.*, **71**, 1–23, 1951, Theorem 2.2.

[4] J. L. Walsh, *J. math. pures et appl.*, **31**, 221–244, 1952, Theorem 1.3. Cf. also I. E. Block, *Duke Math. J.*, **19**, 367–378, 1952.

[5] J. L. Walsh, *Trans. Am. Math. Soc.*, **73**, 447–458, 1952, Theorem 4. Cf. also Elliott, *Proc. Am. Math. Soc.*, **4**, 161–167, 1953.

[6] J. L. Walsh and H. M. Elliott, these PROCEEDINGS, **38**, 1058–1066, 1952, Theorem 1.

*SIMPLIFIED MODEL TO DEMONSTRATE THE ENERGY FLOW AND FORMATION OF TRAVELING WAVES SIMILAR TO THOSE FOUND IN THE COCHLEA**

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The hearing nerves of vertebrates have their endings on the basilar membrane. The total length of this membrane in man is 35 mm. Its width varies continuously from 0.04 to 0.5 mm., and accordingly the stiffness of the membrane decreases over its length one hundred fold. The whole membrane is imbedded in fluid, and, when it is set in vibration, waves travel from the stiff part of the membrane toward the softer part. The properties of these waves are of major interest.

Our concepts of waves have developed mainly from our experience with waves on the surface of fluids and waves of light, where the so-called geometric rules of optics seem to be particularly applicable. Ordinarily the waves we deal with are propagated in or along a homogeneous medium. In the basilar membrane, however, the waves travel along a medium which changes its physical properties continuously. Experiments show that under these conditions the familiar rules of wave propagation may lead to some wrong conclusions.

Let us consider, for instance, a well-known demonstration experiment, used to show the diffusion between two fluids. As can be seen in Figure 1, a concentrated copper sulfate solution is placed in the bottom of a plane-parallel glass trough, and on top of this a layer of plain water is floated slowly in order to avoid any turbulence. In a short time the copper sulfate diffuses into the water, and the color of the mixture shows continuous transition. If a small horizontal light beam enters the trough close to the bottom, it crosses the trough without any change in its

direction of travel. But if the light beam enters the trough at a level at which diffusion is taking place, then, as Figure 1 shows, the light beam is deflected downward,[1] in spite of the fact that the index of refraction in the horizontal plane is constant. The deflection increases with the gradient of the *vertical* change in the refractive index. Thus the simplified geometric concept of wave propagation is not applicable.

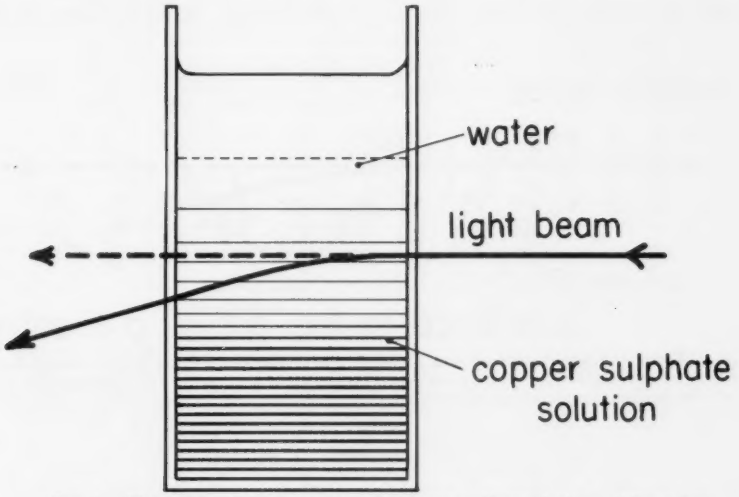

FIG. 1.—Plain water is floated on top of a copper sulfate solution. After a few minutes, because of the diffusion, a continuous transition occurs between the two fluids. If a small light beam travels in the horizontal plane parallel to the layers, the light beam is deflected toward the solution with the higher refractive index. This shows that the approach of geometric optics is not always useful.

A similar phenomenon occurs in the propagation of sound waves above the heads of the audience in a concert hall. It is interesting to rise from one's seat on the floor of a concert hall during a concert. About 50 cm. above the heads of the audience, the loudness of the music increases noticeably, in spite of the fact that there is no visible obstacle between the performer and the seated listener. It is possible to show experimentally that the mere presence of an absorbing layer parallel to the pathway of the sound decreases the amplitudes of the sound waves[2] and deflects them from their straight propagation[3] (see Fig. 2).

In both these examples the continuous change in the medium is perpendicular to the direction of propagation of the waves. In the basilar membrane, however, the continuous change takes place along the direction in which the waves travel, and under these circumstances the phenomena are even more complicated. A schematic drawing of the cochlea is presented in Figure 3. The alternating air pressure in the meatus sets the eardrum, and then the stapes footplate into vibration. These vibrations are transmitted to the fluid that surrounds the basilar membrane. Under stroboscopic illumination the traveling waves of the basilar membrane can be observed[4] on living guinea pigs and on preparations of the human temporal bone and can be calculated.[5] The question that concerns us here is: Are

pathological changes in the basilar membrane apt to cause changes in the traveling waves? For instance, if the part of the basilar membrane close to the stapes footplate becomes solidified, how will this affect the movements at the other end of the basilar membrane? According to Helmholtz' conception, each section of the basilar membrane moves fairly independently, without strong coupling with its neighbors. Each section is assumed to receive its energy directly from the fluid pressure acting on it, and solidification of one part of the membrane would not have a very great effect on the movements of another part. If, on the other hand, the

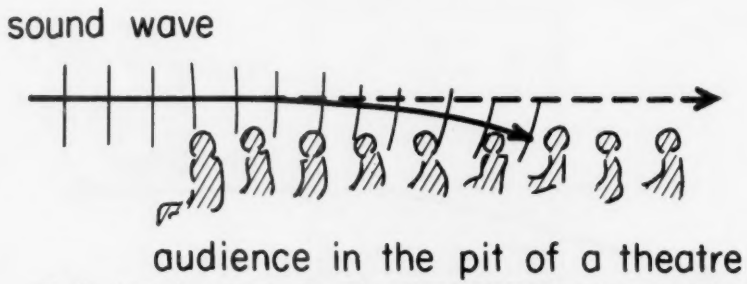

FIG. 2.—A sound wave grazing the surface of an absorbent layer is bent toward the layer and absorbed. This shows again, as in Fig. 1, that waves traveling parallel to the surface of a changing medium are deflected from their straight path.

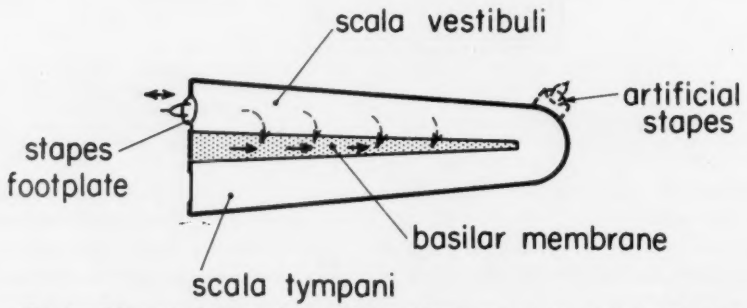

FIG. 3.—Schematic drawing of the cochlea. Movements of the stapes footplate set the fluid column of the scala vestibuli in motion which is transmitted to the basilar membrane. The question is: Which of the two driving forces is dominant on a section of the basilar membrane—the fluid pressure in the scala tympani (*dotted arrows*) or the coupling forces of adjacent parts of the membrane (*solid arrows*)?

coupling between the adjacent parts of the basilar membrane is strong, then we should expect a traveling wave to occur, analogous to the waves observed on ropes. In this case, if the wave were stopped at any point along the basilar membrane, no further waves would be propagated. E. G. Wever and M. Lawrence[6] were interested in this question, and they put it in this form: Does one section of the basilar membrane receive its energy from the surrounding vibrating fluid or from its neighboring section of membrane? In Figure 3 these two paths of energy uptake are represented by solid and dotted arrows.

In order to investigate in detail the ramifications of this problem, a mechanical model has been constructed, in which the movements are large enough and slow enough to be observed by the naked eye.

Description of the Model.—A sudden displacement of the stapes footplate in the direction of the helicotrema (to the right in Fig. 3) produces a compression wave in the vestibular fluid column of the cochlea, and this wave travels away from the stapes. This driving pressure wave produces a pressure difference between the vestibular and tympanic canals of the cochlea, which in its turn sets the basilar membrane in motion and produces a transverse wave in the basilar membrane. As has been pointed out earlier, these two waves are present simultaneously.[7] On the model illustrated in Figure 4, the compression wave in the scala vestibuli is

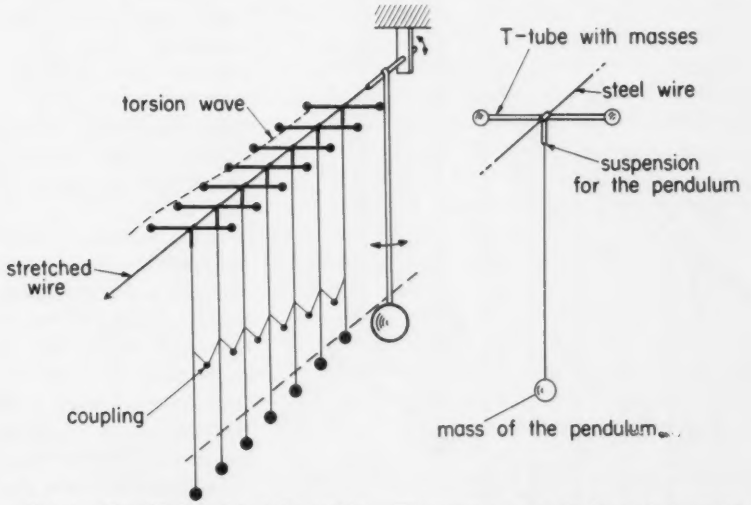

FIG. 4.—Simplified model of the cochlea. The pressure wave in the scala tympani of Fig. 3 is represented by a stretched wire bearing masses. The pendulums are suspended on small vertical arms, so that a torsion wave traveling along the wire sets the pendulums in motion. The pendulums increase in length from right to left and are coupled by small balls. When the torsion waves travel from right to left, very much slower traveling waves occur on the pendulums.

represented by a torsion wave along a stretched steel wire. As can be seen on the right of the drawing, brass T-shaped tubes with small balls on the ends of the two arms were attached to the wire. By increasing the mass of the balls and the length of the arms, it is possible to slow down the speed of the torsion waves until they can be followed by the naked eye. The basilar membrane with its continuously varying stiffness is represented in the model by a set of pendulums, continuously graduated in length and suspended from the vertical branch of the T-tube. The pendulums can swing completely free, or each can be coupled to its neighboring pendulum by means of small balls suspended from threads between the pendulums (see Fig. 4). When the coupling balls are close to the masses of the pendulums the coupling is strong, and when they are close to the suspension the coupling is weak.

A torsion wave along the stretched wire displaces the suspension and sets the pendulums in motion. Increasing the distance between the point of suspension and the axis of the wire increases the driving force acting on the pendulums. Torsion waves can be produced in the wire either by a heavy driving pendulum or by an eccentric wheel moved by a synchronous motor at a constant speed.

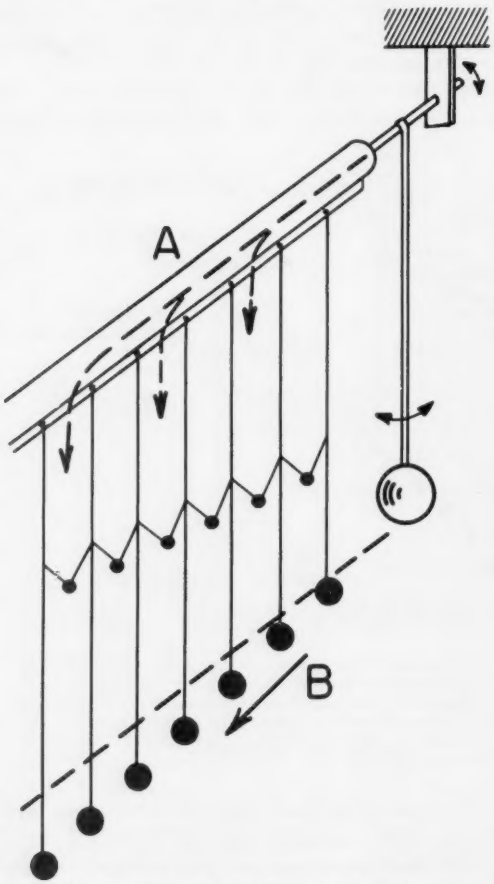

FIG. 5.—Since the velocity of the compression wave is at least 20 times larger in the vestibular fluid than along the basilar membrane, the model in Fig. 4 was further simplified by making the velocity of the torsion waves infinite; this was accomplished by means of a tube oscillating around its axis which served as the driving force.

In this kind of model two waves can be seen, the driving torsion waves of the wire and the waves of the pendulums produced by the torsion wave. The torsion waves have a constant wave length, but the pendulum waves are longer near the short pendulums and shorter near the long pendulums where the coupling is smaller. Both waves travel away from the driving pendulum.

In the cochlea itself, the speed of the driving compression waves is very much higher than the speed of the waves observed along the basilar membrane. At 1,000 cps the ratio in the human cochlea is more than 20:1. Accordingly, the speed of the torsion waves of the model has been made infinitely large by substituting for the stretched wire a solid cylinder oscillating around the axis of the wire (see Fig. 5). With this change the driving force of the pendulums has the same magnitude and phase along the whole line of pendulums. In Figure 5 the dotted arrows represent the flow of energy from the fluid of the scala tympani directly to the different sections of the basilar membrane, and the solid arrow represents the flow of energy along the basilar membrane.

The specifications of the model are as follows: number of pendulums, 48; length of shortest pendulum, 43 cm.; length of longest pendulum, 53 cm.; distance of suspension from axis of driving rod, 4.5 cm.; period of driving force, $T = 1.35$

seconds; mass of the pendulums, 35 gm.; mass of the coupling balls, 3.5 gm.; height of the coupling balls above the shortest pendulum, 3 cm.; height of the coupling balls above the longest pendulum, 35 cm.; distance between the pendulums, 5 cm.

Traveling Waves in a System of Pendulums with No Coupling.—Let us first consider the case in which there is no energy transmission from one pendulum to its neighbor because there is no coupling between them. This situation can be obtained on the model by raising the suspensions of the coupling balls until they reach the suspension point of the pendulums. The system now consists of a series of free pendulums of continuously increasing length.

If the driving axis is suddenly rotated a few degrees, all the pendulums are activated at the same moment, and they start to swing. Since each pendulum has a different period of oscillation, a phase difference soon develops, and the pendulums are no longer in a straight line. With time this phase difference increases, and we have the impression of waves that travel from the shorter pendulums to the longer ones. The striking phenomenon is that the wave length decreases continuously with time. Figure 6 shows a series of photographs taken at 10-second intervals from a motion picture. Shortly after the pendulums receive one push, only one half-wave can be seen, but after 50 seconds, five full waves are visible. It is surprising to notice that when the waves reach the end of the pendulum series there is no reflection at all. But this agrees with the fact that when there is no coupling, no energy is transmitted from one pendulum to its neighbor.

This shortening of the wave length with time after a sudden push is not observed in the cochlea. Certainly the damping in the cochlea is larger than in the model; nevertheless, when the basilar membrane is plucked, two full periods of vibrations can be observed on an oscilloscope which records the aural microphonics.[8] This would be time enough to observe a decrease in wave length if there were one. From this we have to conclude that there is an appreciable amount of coupling in the basilar membrane.

Generation of Waves.—The waves are produced by the phase difference between adjacent pendulums (Fig. 6). If all the pendulums are of the same length, there is no phase difference between them, and, as can be seen in Figure 7, they merely swing back and forth in a line, irrespective of the height of the coupling balls. The waves originate only at the place where a change in the length of the pendulums occurs. In order to show this more distinctly, the right-hand half of the pendulums were made a constant short length and the left-hand half were made a constant longer length (Fig. 8). The driving rod started to oscillate at the moment $t = 0$, and, as can be seen from the prints made from a motion picture, two waves are formed which travel away from the discontinuity point, the one traveling to the right with a long wave length and the other to the left with a very much shorter wave length, mainly because the coupling between the pendulums is much less in the left section than in the right section. Both waves show a well-circumscribed front which travels with time away from the discontinuity point.

I think that it is important to an understanding of the different types of wave formation to start with the picture that, wherever a change in the stiffness occurs, two waves are originated, one traveling to the left side, the other to the right. When there are many changes along the system, they add together and form the final visible wave. When the lengths of the pendulums increase continuously,

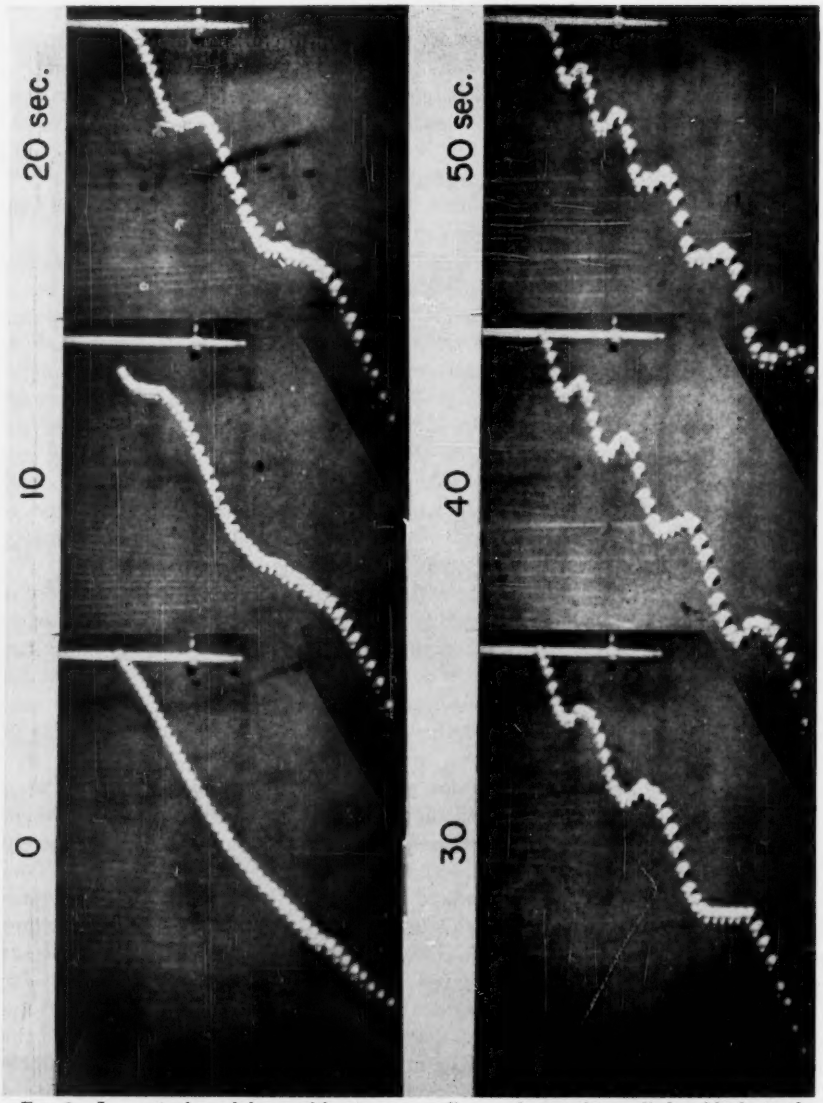

FIG. 6.—In a set of pendulums without any coupling, a short twist applied suddenly to the driving rod produces waves whose length shortens with time. No reflection occurs at the end of the pendulum series.

each section contributes to the formation of traveling waves. The final wave is illustrated in Figure 9, where the wave moves from right to left and reaches its maximum amplitude in the middle of the series. These pictures are taken at $^1/_6$-second intervals during steady continuous oscillation. They are quite similar to

the wave pattern observed in the cochlea. If the frequency of the oscillations is increased, the maximum of the amplitudes moves to the right, thus transforming the frequency change into a displacement of the place of maximal amplitude. Since the pendulum series is relatively short, reflection cannot be avoided at both ends of the series, and standing waves may occur on the model.

If we now go back to Figure 8, we can see that every change in the stiffness spreads out the energy along the series of pendulums. Since this spreading takes the form of waves, the energy may move far away if the damping of the waves is not too strong.

Energy Transmission to the Different Parts of the Membrane.—From the previous discussion it is seen that a pendulum receives part of its energy from the oscillating driving rod directly (dotted arrows in Fig. 5) and part from its neighboring pendulums (solid arrows). The question becomes, therefore, which part is the larger one? On the model the movements of all the longer pendulums can be stopped by

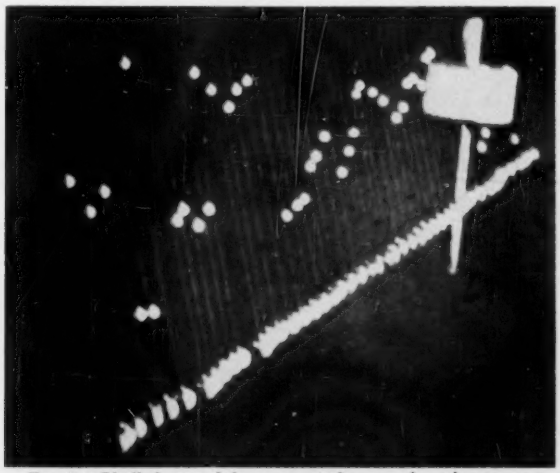

FIG. 7.—If all the pendulums are of the same length, no waves can be seen. In this and all the following figures there was coupling between the pendulums.

pressing against them a wooden bar covered with foam rubber, as shown in Figure 10. When this is done, the amplitudes of the movements of the shorter pendulums are not, in general, diminished but are actually increased, since a standing wave is formed. From this we must conclude that the shorter pendulums receive their energy from the oscillating rod and not from the longer pendulums. On the other hand, if we stop the movements of the shorter pendulums, as in Figure 11, the amplitudes of the longer pendulums are decreased. This can be seen clearly if we compare Figure 11 with Figure 9, which shows the amplitude distribution of the undisturbed waves for the same oscillation amplitudes of the driving rod. This indicates that the flow of energy from the shorter pendulums toward the longer ones is dominant for those pendulums whose amplitude is maximum for a particular driving frequency. An even more convincing experiment is to let the system

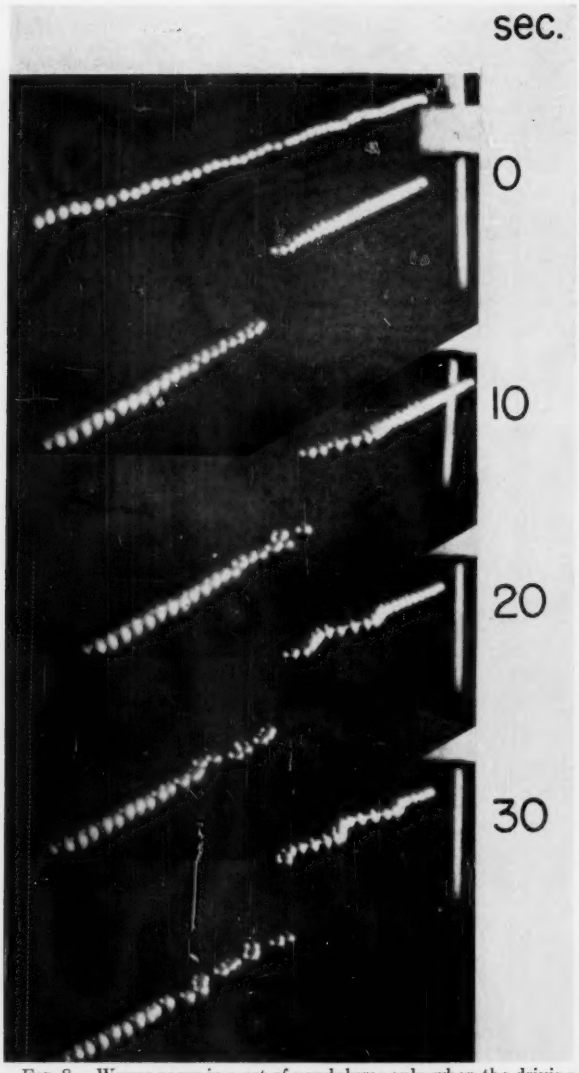

FIG. 8.—Waves occur in a set of pendulums only when the driving forces are continuous sinusoidal oscillations at the place where a change occurs in the lengths of the pendulums. From the place of change, the waves travel in both directions.

with the stopped shorter pendulums oscillate until a steady state occurs. Then, if the damping foam-rubber board is removed abruptly, the shorter pendulums start to oscillate. When the waves thus formed reach the longer pendulums, they are in opposite phase to the waves formed by the longer pendulums. The result is that the waves cancel each other out, and the amplitudes of the longer pendulums

FIG. 9.—When the length of the pendulums changes continuously, the small waves add together, and, if the driving force consists in continuous sinusoidal oscillations, a traveling wave forms which has a maximum at one place. If the driving frequency is changed, this maximum is displaced.

are further diminished. It takes some time before the phase is reversed and the larger amplitude is finally obtained.

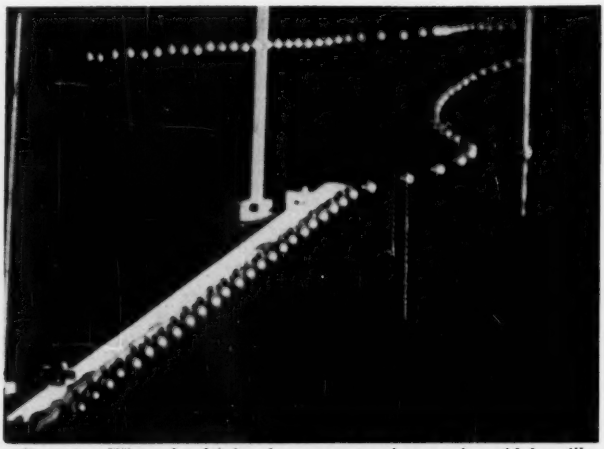

FIG. 10.—When the driving forces are continuous sinusoidal oscillations, stopping the long pendulums does not diminish the amplitudes of the short pendulums. This indicates that the short pendulums receive their energy directly from the driving rod.

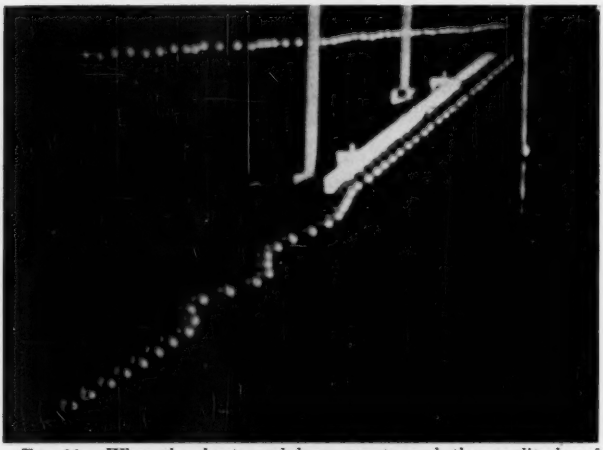

FIG. 11.—When the short pendulums are stopped, the amplitudes of the longer pendulums are diminished. Thus it is likely that the longer pendulums receive part of their energy from the shorter pendulums.

The result is similar if we simply stop one pendulum in the area of maximal amplitude. The amplitudes of the shorter pendulums increase, and those of the longer pendulums decrease.

Still another way to show that the energy of a pendulum tends to flow in the direction of the longer pendulums is to give a small section of the pendulums a push

and then to observe the direction of the waves thus produced. The ten longest pendulums were displaced slightly and then suddenly released. As can be seen in Figure 12, the potential energy of this section did not produce a wave of large amplitude traveling toward the shorter pendulums, but, as the picture taken 20 seconds after the release shows, the longest pendulum has the largest excursion.

If the damping of the pendulums is increased by letting them swing in water or oil, the energy transmitted from one pendulum to its neighbor soon diminishes. But the general trend is still the same, and when the coupling between the pendulums is increased, the wave length increases, and with it the distance the energy travels.

Paradoxical Waves.—The tendency of energy to flow from the shorter to the longer pendulums as a consequence of the coupling leads to some surprising phenomena. In the experiments described above, the length of the driving lever (the distance between the axis of the driving rod and the suspension in Fig. 4) was the same for all the pendulums. It is possible, however, to increase the length of the driving lever of the longer pendulums, relative to the shorter ones, without changing the direction the waves travel. In this case the traveling waves move toward the place where the driving amplitude is higher. This is a startling phenomenon, since we are accustomed to seeing waves move away from the source of sound or vibration. In the experiment illustrated in Figure 13, the same set of pendulums was

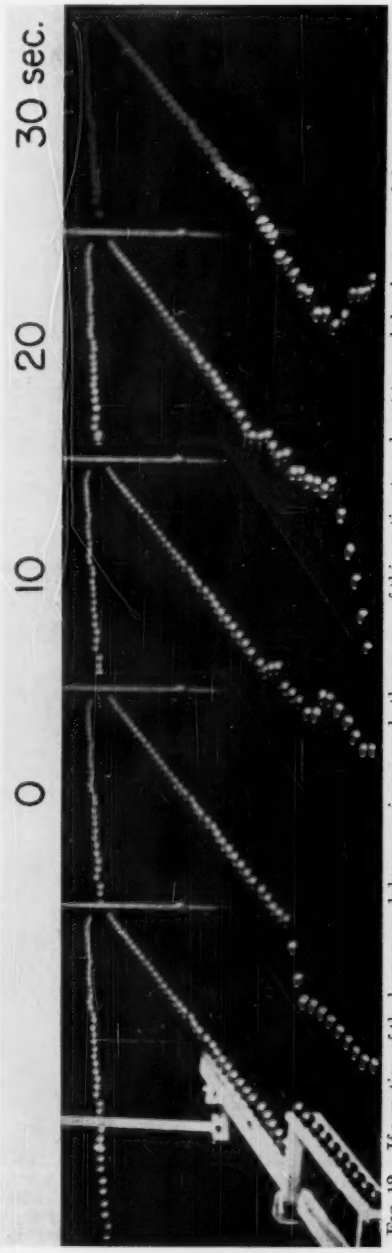

FIG. 12.—If a section of the longer pendulums receives a push, the energy of this section travels not toward the shorter pendulums but almost exclusively toward the longer ones. This is another illustration of the tendency of the energy of the pendulums to flow, via the coupling, from the short to the long pendulums.

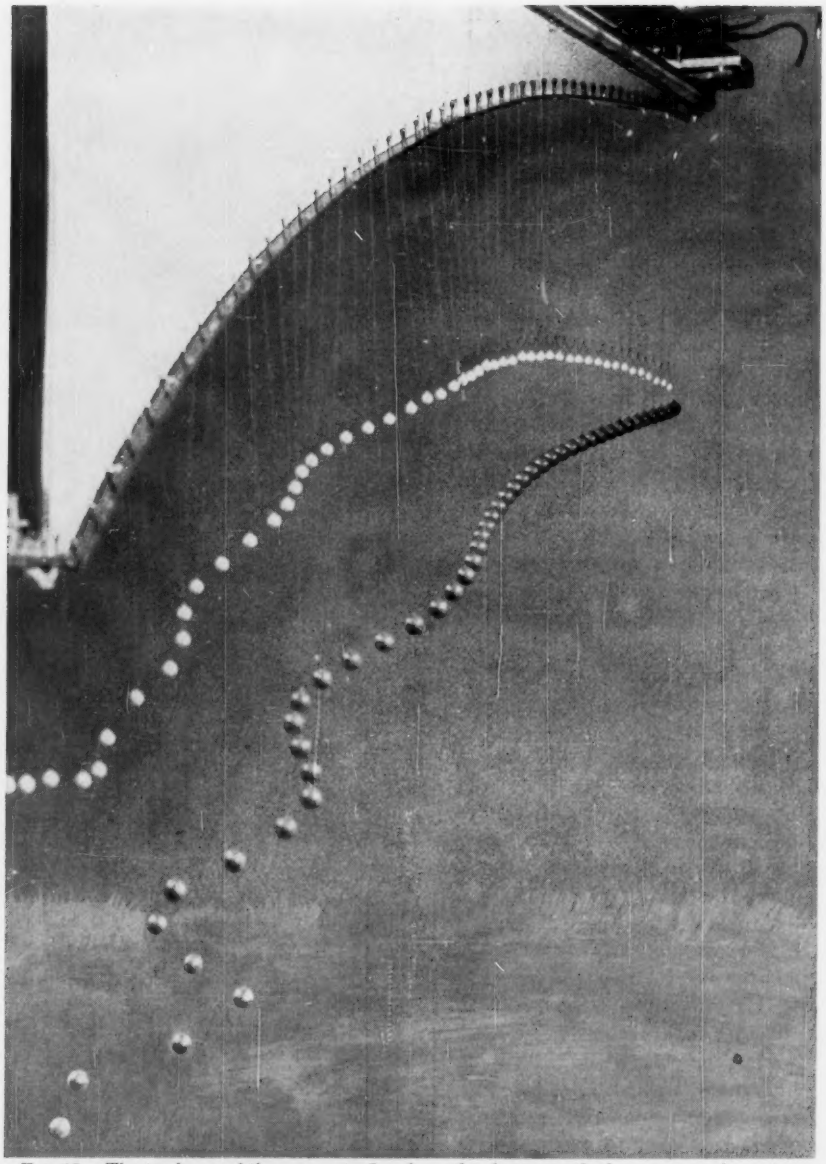

FIG. 13.—The tendency of the energy to flow from the shorter to the longer waves is so strong that the direction the waves travel remains unchanged, even when the driving forces are made 16 times larger for the long pendulums than for the short pendulums. The waves travel from right to left against the strong driving forces acting on the left.

fixed in an arched curve on the oscillating driving rod, in such a way that the force acting on the pendulums doubled in size every twelfth pendulum, counting from the shortest pendulum. Since the model contained 48 pendulums, the driving force acting on the longest pendulum was 16 times larger than the driving force acting on the shortest pendulum. Although a very large amount of energy was transmitted directly from the oscillating suspension to the long pendulums, the waves still traveled in the same direction as before, i.e., from the shorter pendulums to the longer ones, and with a very similar pattern and only a slight displacement of the place of maximal amplitude. Just watching the waves, I had the impression that it would take a very much larger increase in the driving force to reverse the direction of the traveling waves.

The model represents only a small section of the actual cochlea. Since this unequal distribution of the driving forces did not change the direction of the traveling waves in the model, it is not surprising that in the cochlea, too, the direction of the traveling waves is to a high degree independent of the location of the driving forces. Actually, this phenomenon was first observed on a preparation of human cochlea.[9] As is shown in Figure 3, substitution for the stapes footplate of an artificial stapes at the other end of the basilar membrane (dotted drawing to the right) changed only the sensitivity but not the direction of the traveling waves, and beyond this, no change was observed under the microscope. E. G. Wever and M. Lawrence,[10] recorded microphonics to indicate the vibrations of the basilar membrane and came to the same conclusion. Two mathematical interpretations of the phenomena have been presented by J. Zwislocki,[11] one using Hamilton's principle. Some experiments designed to explain this phenomenon have been reported in an earlier publication.[12]

Wave Patterns Corresponding to Pathological Cases.—The pendulum model is especially suited to the investigation of the effect of local disturbances, such as those found in pathological cases, on the general wave pattern. As Figure 10 shows, stopping a large section of the longer pendulums (analogous to the area near the helicotrema) does not decrease the vibration amplitude of the shorter pendulums (near the stapes). But if a section of the shorter pendulums is stopped as shown in Figure 11, the vibration amplitude decreases to about one-third its earlier value. The largest changes in amplitude occur when a continuous sinusoidal driving force is used and the pendulum with maximal amplitude is stopped. This is shown in Figure 11, where the movement of all the pendulums to the right of the place of maximal amplitude was arrested. If only a small section of pendulums at the extreme right is stopped, the effect is smaller, unless phase reversal causes a cancellation of the waves to occur. If the same section of pendulums is stopped, but the driving frequency is changed, we may find that at some frequencies the cancellation is maximal and the vibration amplitude minimal. But a small frequency change may increase the amplitudes above their normal value. It is possible, therefore, that abrupt changes in the threshold of hearing with frequency may be due to such a situation. Until now, these changes in frequency sensitivity have been attributed to sudden changes in the transmission characteristics of the middle ear.

In order to evaluate the observations made on the model, it is important to remember that an amplitude decrease of 3:1 equals a change in threshold of only 10 db. Clinical threshold measurements are seldom more precise than ±5 db. This fact limits the safety of speculations.

Summary.—The propagation in a medium with continuously changing properties shows some little-known effects. If a sound wave is propagated parallel to a sound-absorbing layer, the sound wave is deflected from its straight path and bent toward the absorbing layer. In this way much absorption may occur, even though there are no obstacles in the path of the sound wave. In order to illustrate what may happen to waves that travel in the direction in which the mechanical properties of the medium are changing continuously, a model consisting of a series of pendulums was used in which the length of the pendulums was continuously graduated. With proper coupling between the pendulums, traveling waves occur which are very similar to those observed in the cochlea of mammals.

On the model it can be shown that every change in mechanical properties produces two waves traveling in opposite directions and that the addition of a series of such waves produces an over-all wave that travels from the stiffer part of the system to the softer part. The direction of travel is maintained even when the oscillatory forces acting on the softer parts are larger than those acting on the stiffer ones: in this case, a wave forms which travels toward the oscillating source.

* This research was carried out at the Psycho-Acoustic Laboratory, Harvard University, under Contract Nonr-1866(15) with the Office of Naval Research, United States Navy (Project NR142-201, Report PNR-187). Reproduction for any purpose of the United States Government is permitted. A motion picture of the traveling waves was shown at the Second International Congress on Acoustics in Boston, June, 1956.

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[9] G. von Békésy, *Physik. Z.*, **29**, 793–810, 1928.

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[11] J. Zwislocki, *J. Acoust. Soc. Amer.*, **25**, 986–989, 1953.

[12] G. von Békésy, *J. Acoust. Soc. Amer.*, **27**, 137–145, 1955.

CURRENT RESEARCH FINDINGS ON RADIOACTIVE FALLOUT

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UNITED STATES ATOMIC ENERGY COMMISSION, WASHINGTON, D.C.

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I. INTRODUCTION

The radioactivity produced by the fission reaction, being due to a mixture of many different fission products, changes its characteristics continuously and rapidly following release by the bomb detonation. Thus the conditions of firing are of extreme importance in determining the fallout effects. The intensity of radiation is enormously greater soon after the detonations, decreasing about tenfold for every seven fold increase in age. Since the time required for ingestion into the body is long, ingestion is unlikely for the shorter-lived fission products, and therefore the principal hazards for close-in fallout are radiation exposures by gamma radiation of the whole body and by beta radiation on the skin.

In the longer times, weeks and months after the explosion, the ingestive hazards begin to become important. The most serious of these is the high-yield fission product, radioactive strontium (Sr^{90}), which because of its own radiation and those of its short-lived daughter, yttrium 90 (Y^{90}), and because of its chemical similarity to the bone-building element, calcium, finds itself deposited in bone structure. Other radioactivities produced would be as bad if they spent as long a time in the body or if their radioactive lifetimes were long enough or if they were produced in high yield. Strontium has all these characteristics. Hence for the fission products which have survived the first weeks, the most important fallout constituent and the one most seriously to be considered is Sr^{90}. Neither radiostrontium nor its yttrium daughter emits gamma radiation, but only beta radiation. After the first-year cesium 137 (Cs^{137}), with half-life of thirty-three years, is the principal source of the residual gamma radiation, and any gamma radiation exposures due to fission products which are more than one year old are due very largely to radioactive cesium. In fact, old fallout can be thought of as a mixture of roughly equal radiation intensities in millicuries of radioactive Sr^{90} and radioactive Cs^{137}. The other isotopes either constitute no ingestive hazard or fail to emit gamma radiation in appreciable intensity. Thus, the hazards of world-wide fallout reduce themselves largely to the ingestive hazard of radioactive Sr^{90} and the external exposure from radioactive Cs^{137}.

The mechanism by which atomic-weapon debris is disseminated leads to three kinds of fallout. First, there is the strictly *local fallout*, which is due to the return to earth of the larger particles in the fireball. These may have their origin in the dirt, the soil, or tower structures which are taken into the fireball and either wholly or partially vaporized. The fraction of the total which falls out locally depends very much upon the conditions of firing. The most serious factor is the degree of contact of the fireball with the surface; another is the nature of the surface. For example, soil appears to be much more effective than water in producing local fallout.

Experience has shown that an atomic device exploded on the surface distributes about 80 per cent of its fission products on the ground within a few hundred miles of the burst point. A somewhat larger percentage takes part in the close-in fallout from an underground burst, and a smaller percentage will be scavenged from a near-surface burst or tower shot.

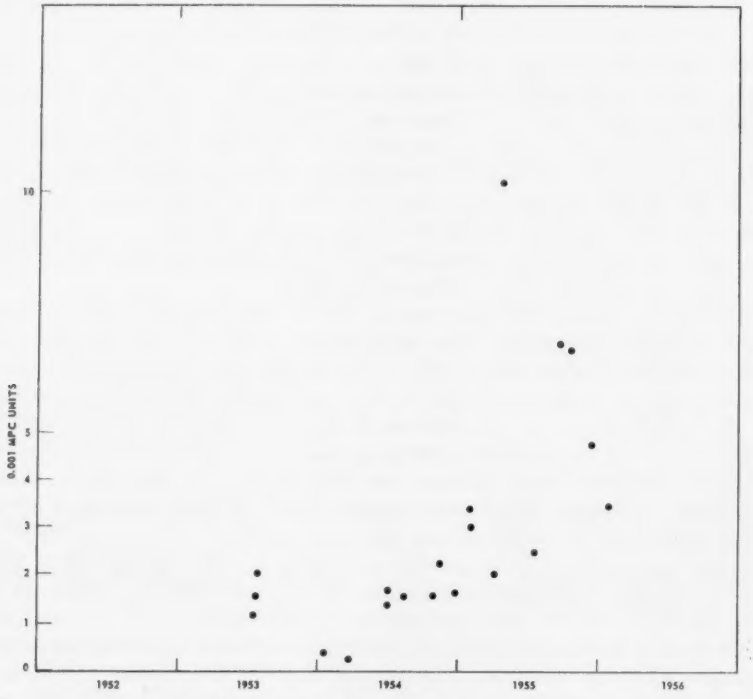

FIG. 1*a*.—Wisconsin cheese, Sr^{90} content

The tower shot is, in a sense, a special case of a surface burst, since the material of the tower itself is mixed with the fission products in the fireball to a greater or lesser degree, depending on the yield. Experience with tower shots indicates that even in cases where the fireball does not touch the ground a few per cent of the radioactive fission products come down as close-in fallout.

The fraction which takes part in the close-in fallout from a surface burst over deep ocean water appears to be somewhere between 20 and 50 per cent. This is less than the fraction of close-in fallout occurring from a corresponding surface burst over land, due to the evaporation of many of the drops before they reach the ground. Presumably this fraction is also affected by the prevailing humidity and temperature structure of the atmosphere through which the drops must fall. As the depth of the water is decreased, the point is reached where the fireball extends downward to the bottom and picks up bottom material. In such shallow water one would expect a higher percentage of close-in fallout than in deep water. Ex-

perience in the Pacific indicates that such is indeed the case and that in fact there would be very little difference in the fallout between a large-yield device in very shallow water and a true surface shot.[1]

The second type of fallout is the material which, though not coarse enough to fall of its own weight in the first few hours, is, nevertheless, left in the lower layer of the atmosphere, known as the *troposphere*, where ordinary weather phenomena occur. It is now well established that fallout particles are removed from this lower layer of the atmosphere in the first month or so; therefore, the tropospheric

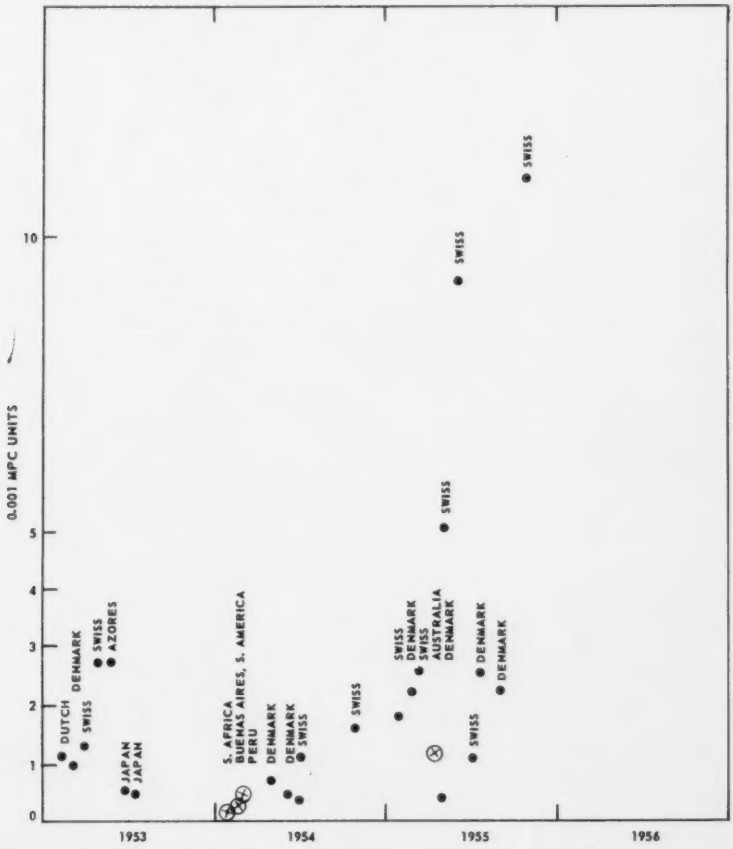

FIG. 1*b*.—Foreign cheese, Sr^{90} content (*crosses:* Southern Hemisphere)

fallout stays airborne at most a matter of a month or two before being deposited. Of course, in this time, it will in general move great distances, possibly even all the way around the earth; but, in general, it stays in the general latitude in which the explosion occurred. So the second type of fallout, the tropospheric world-wide fallout, produces a band of radioactivity in the general latitude of the firing site.

The fraction of the fallout in this category depends mainly on the bomb yield and the conditions of firing. A bomb which is fired on the ground produces a maximum of *local fallout*, naturally leaving less for the world-wide fallout of either the tropospheric or stratospheric variety. The bomb yield determines the division of the world-wide fallout between the two kinds of world-wide fallout. A general rough rule is that a 1-megaton bomb will produce clouds which push into the higher layer of the atmosphere, the stratosphere, before disseminating and that the clouds from bombs of less than 1 megaton will tend to stay mainly in the troposphere. Thus we see that a 500-kiloton weapon fired so its fireball did not touch the ground would

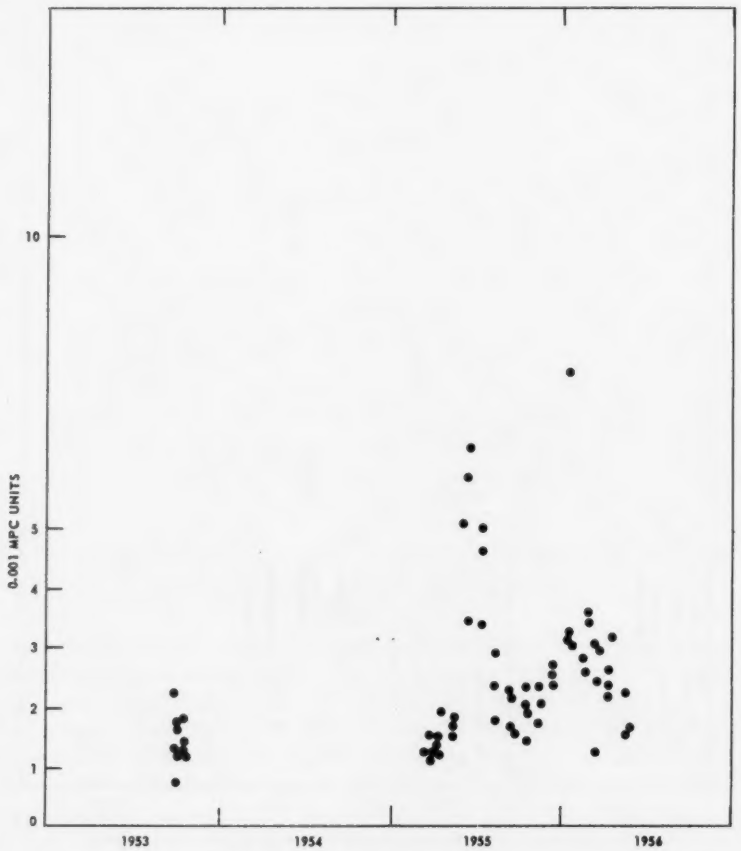

FIG. 1c.—Chicago Milk, Sr^{90} content

be expected to put the major part of its radioactivity in a band stretching around the world in the general latitude of the firing site. The distribution of the activity would be world wide and would be completed within the first month or two. Similarly, the same bomb fired in contact with the earth with ordinary soil would have a

large fraction—something like 80 per cent—of its fallout deposited within the first few hours within a few hundred miles of the test site, and the rest of the material would be spread in the tropospheric world-wide fallout pattern in a band around the earth in the same general latitude.

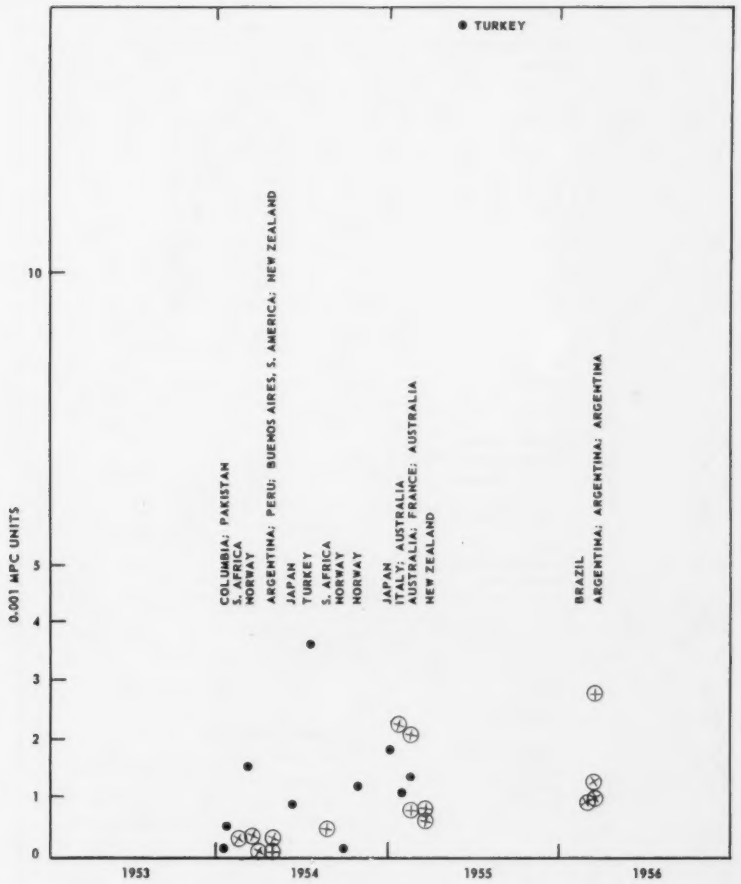

FIG. 1*d*.—Foreign milk, Sr^{90} content (*crosses:* Southern Hemisphere)

The third type of fallout is the *stratospheric world-wide fallout.* Weapons of yields of 1 megaton and greater thrust their radioactive clouds into the stratosphere, and the material which does not fall of its own weight within the first few hours is then very largely borne in the stratosphere for great lengths of time. An average time seems to be about ten years or somewhat less. A small amount of tropospheric world fallout is also produced, presumably due to a small fraction of particulate matter which is of just the right size to descend in a matter of weeks. The division into the two types, the *local* and the *stratospheric world-wide*, is very sharp and

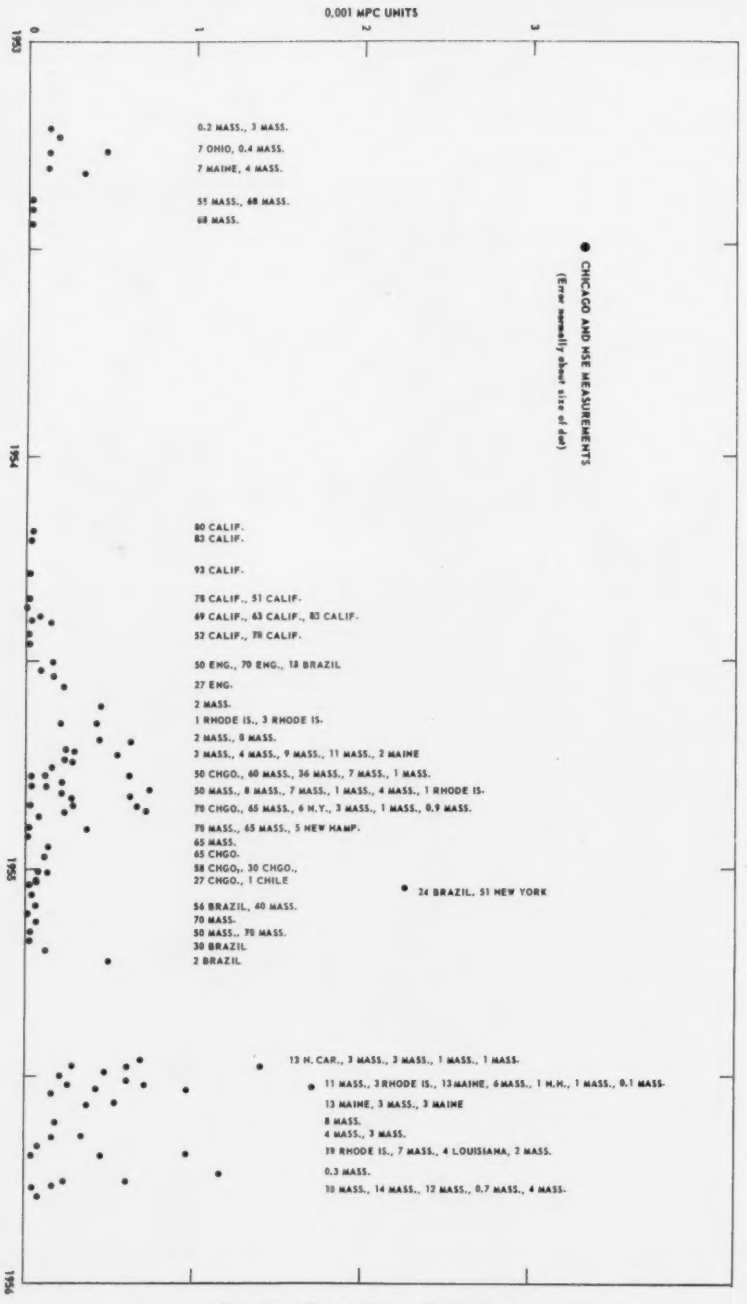
0.001 MPC UNITS
0
1
2
3
1953
1954
1955
1956
0.2 MASS., 3 MASS.
7 OHIO, 0.4 MASS.
7 MAINE, 4 MASS.
55 MASS., 68 MASS.
68 MASS.
CHICAGO AND HSE MEASUREMENTS
(Error normally about size of dot)
80 CALIF.
83 CALIF.
93 CALIF.
78 CALIF., 51 CALIF.
69 CALIF., 63 CALIF., 83 CALIF.
52 CALIF., 78 CALIF.
50 ENG., 70 ENG., 18 BRAZIL
27 ENG.
2 MASS.
1 RHODE IS., 3 RHODE IS.
2 MASS., 8 MASS.
3 MASS., 4 MASS., 9 MASS., 11 MASS., 2 MAINE
50 CHGO., 60 MASS., 36 MASS., 7 MASS., 1 MASS.
50 MASS., 8 MASS., 7 MASS., 1 MASS., 4 MASS., 1 RHODE IS.
70 CHGO., 65 MASS., 6 N.Y., 3 MASS., 1 MASS., 0.9 MASS.
70 MASS., 65 MASS., 5 NEW HAMP.
65 MASS.
65 CHGO.
58 CHGO., 30 CHGO.,
27 CHGO., 1 CHILE
24 BRAZIL, 51 NEW YORK
56 BRAZIL, 40 MASS.
70 MASS.
50 MASS., 70 MASS.
30 BRAZIL
2 BRAZIL
13 N. CAR., 3 MASS., 3 MASS., 1 MASS., 1 MASS.
11 MASS., 3 RHODE IS., 13 MAINE, 6 MASS., 1 N.H., 1 MASS., 0.1 MASS.
13 MAINE, 3 MASS., 3 MAINE
8 MASS.
4 MASS., 3 MASS.
19 RHODE IS., 7 MASS., 4 LOUISIANA, 2 MASS.
0.3 MASS.
10 MASS., 14 MASS., 12 MASS., 0.7 MASS., 4 MASS.

FIG. 2*a*.—Human bone, Sr^{90} content

marked, however, and to a very considerable approximation one can say that the megaton weapons yield the bulk of their fallout in these two categories. A weapon involving 1 megaton of fission would, if fired in the air, place most of its radioactivity in the stratosphere, and this, in contrast to the tropospheric fallout, appears to be

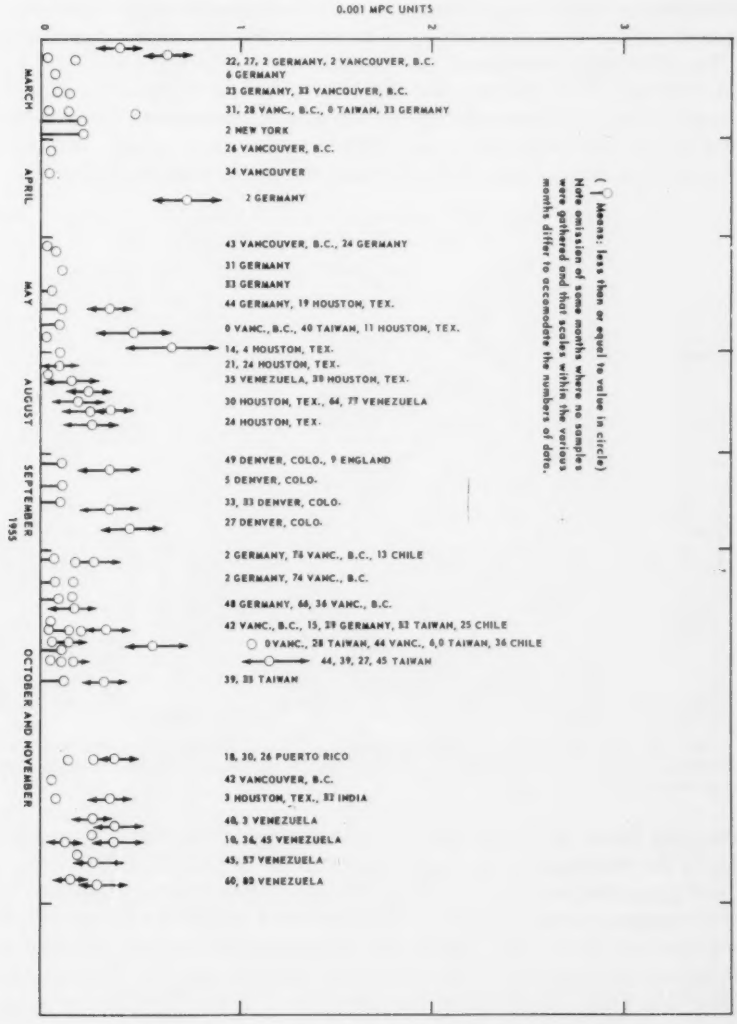
0.001 MPC UNITS
0
1
2
3
MARCH
APRIL
MAY
AUGUST
SEPTEMBER
1955
OCTOBER AND NOVEMBER
22, 27, 2 GERMANY, 2 VANCOUVER, B.C.
6 GERMANY
23 GERMANY, 33 VANCOUVER, B.C.
31, 28 VANC., B.C., 0 TAIWAN, 33 GERMANY
2 NEW YORK
26 VANCOUVER, B.C.
34 VANCOUVER
2 GERMANY
43 VANCOUVER, B.C., 24 GERMANY
31 GERMANY
33 GERMANY
44 GERMANY, 19 HOUSTON, TEX.
0 VANC., B.C., 40 TAIWAN, 11 HOUSTON, TEX.
14, 4 HOUSTON, TEX.
21, 24 HOUSTON, TEX.
35 VENEZUELA, 38 HOUSTON, TEX.
30 HOUSTON, TEX., 44, 77 VENEZUELA
24 HOUSTON, TEX.
49 DENVER, COLO., 9 ENGLAND
5 DENVER, COLO.
33, 33 DENVER, COLO.
27 DENVER, COLO.
2 GERMANY, 74 VANC., B.C., 13 CHILE
2 GERMANY, 74 VANC., B.C.
48 GERMANY, 66, 36 VANC., B.C.
42 VANC., B.C., 15, 39 GERMANY, 33 TAIWAN, 25 CHILE
0 VANC., 28 TAIWAN, 44 VANC., 6,0 TAIWAN, 36 CHILE
44, 39, 27, 45 TAIWAN
39, 33 TAIWAN
18, 30, 26 PUERTO RICO
42 VANCOUVER, B.C.
3 HOUSTON, TEX., 32 INDIA
48, 3 VENEZUELA
10, 36, 45 VENEZUELA
45, 57 VENEZUELA
60, 89 VENEZUELA
(Means: less than or equal to value in circle)
Note omission of some months where no samples were gathered and that scales within the various months differ to accomodate the numbers of data.

FIG. 2*b*.—Human bone, Sr^{90} content, Lamont measurements

widely distributed latitudinally and does not descend through the tropopause into the troposphere until months and years—an average time of about ten years—have elapsed. The resultant pattern of fallout appears to be essentially in approximately uniform world-wide distribution. The long time spent in the stratosphere is proba-

bly due to the absence of such scavenging agents as rain or snow, so the particles either must fall out of their own weight or diffuse downward by slight eddy motion, either of these processes being of their very nature slow. After passing through the tropopause into the troposphere, they will be rained out rather quickly in a matter of days or weeks. Because of the long residence time in the air, this type of fallout is particularly harmless as a gamma-ray hazard, since only the Cs^{137} is left. The amount of Cs^{137} is about the same in millicuries as the Sr^{90}; thus 1 megaton of fission thus distributed throughout the stratosphere would yield about 1/2 mc/mi² of either Sr^{90} or Cs^{137}. Just as in the case of Sr^{90}, the rate of deposition of about 10 per cent of the reservoir per year corresponds to a stratospheric fallout rate of Cs^{137} of 0.05 mc/mi²/yr in the beginning. This rate decreases to half, or 0.025 mc/mi²/yr, at about seven years, as the stratospheric reservoir becomes depleted.

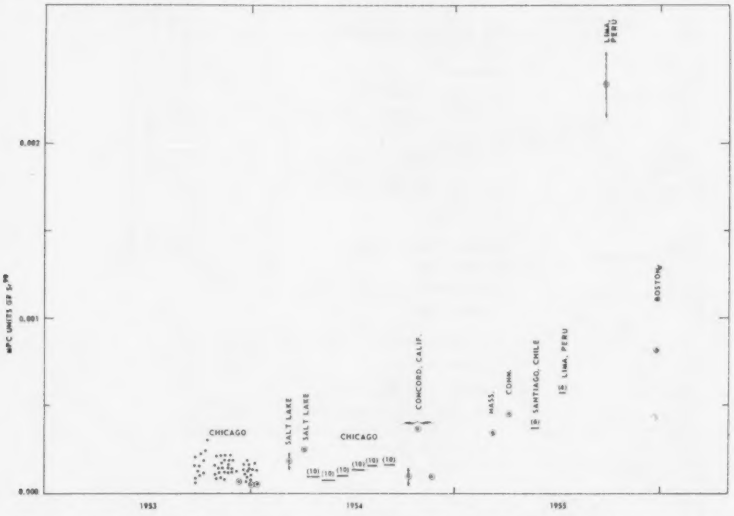

FIG. 2c.—Human stillborns (measurements by Chicago laboratories) (error less than 0.00005, unless otherwise indicated; parenthetical numbers are number of cases used for the average)

After the Castle test series was completed, there were about 24 megatons of fission in the stratosphere, corresponding to about 12 mc/mi² of Sr^{90} on the average and about the same amount of Cs^{137}. The subsequent stratospheric worldwide fallout rate appears to have been a little over 1 mc/mi²/yr all over the world. In addition, of course, local fallout and tropospheric latitudinally localized worldwide fallout have occurred from subsequent weapons tests by the Russians and the Redwing series completed last summer at the Eniwetok Proving Grounds.

II. RESEARCH FINDINGS

A. Amount of Fallout and Expected Sr^{90} Body Burden from Weapons Fired to Date.—Since our last report,[2] further data on the actual magnitude of fallout in various places in the world and in various selected spots in the food chain have be-

come available. We shall summarize the results. Some human bone now contains radioactive strontium at levels of about one-thousandth of maximum permissible concentration (0.001 MPC; the MPC is 1 microcurie of Sr^{90} for the standard man and proportionally less for children, and is the maximum permissible concentration) in the northern latitudes where the bombs have been fired and the world-wide tropospheric fallout has occurred. There is evidence in the data on human material that age is a factor, e.g., older people who have had their calcium deposited

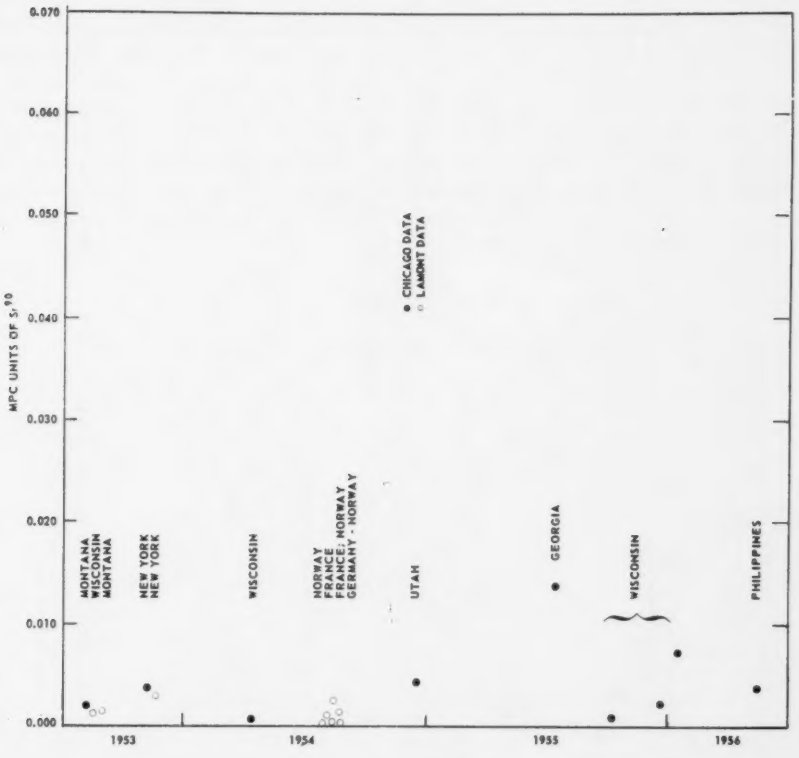

FIG. 3*a*.—Calf bones

prior to the weapons tests show lower concentrations, though in some instances exceptions to this rule are to be found. Lower levels are found in the Southern Hemisphere, where the major contamination is due solely to the world-wide stratospheric fallout, which in the northern latitudes of 10°–50° N. is generally less than one-half or one-quarter of the total fallout. The deposition in the human body seems roughly to parallel the levels of the total fallout. More data are necessary to validate this point fully.

At the end of 1955 the total deposition in the upper Midwest of the United States was some 13 mc/mi² of Sr^{90}. In the spring of 1956 this total rose to about 16 mc/mi².

Between May 5 and mid-July of this year, Operation Redwing was conducted at the Eniwetok Proving Grounds in the Pacific. Particular attention was paid to the fallout problem in this operation, and a major effect was made to produce a megaton-range weapon with an inherently smaller amount of fallout for a given energy release. This effort was successful. In addition, considerable attention

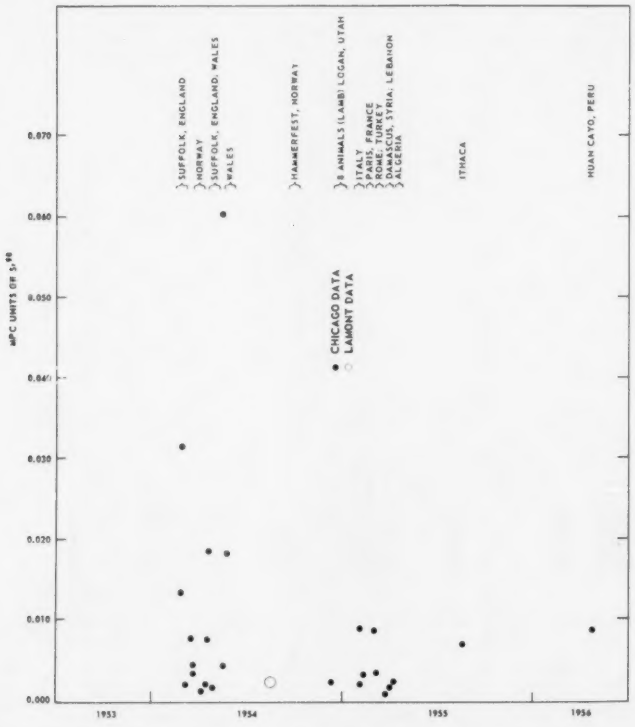

FIG. 3*b*.—Sheep bones

was paid to operational factors which would minimize world-wide fallout. Thus the total deposition in the stratosphere during this operation was held to a figure very considerably less than that present in the stratosphere before the operation. In fact, we estimate at the present time that the total stratospheric reservoir, counting all sources, is about the same as it was two years ago, i.e., 12 mc/mi² or Sr^{90}, or the equivalent of 24 megatons of fission, calculated as a uniform world-wide distribution. During the last two years the additional depositions in the stratosphere have amounted to about 6 megatons equivalent of fission products total, or 3 mc/mi² of Sr^{90} or Cs^{137}. This appears to have compensated approximately for the 10 per

cent per year of fallout and the 2.5 per cent per year of radioactive decay. In other words, the testing by all countries seems to have restored the stratospheric reservoir to approximately the 24-megaton value of two years ago.

The latitudinal tropospheric world-wide fallout, which is maximized by weapons of high yield which do not puncture into the stratosphere, is increasing. Several such weapons have been air-fired abroad in the last months. This material, for the reasons explained above, descends rather rapidly but all the way around the world in the same general latitude as the firing site. Thus, though it is difficult to estimate, it appears that this amounts to perhaps 5 additional mc/mi² of Sr^{90} and Cs^{137} in the United States. Adding to this about 1/2 mc/mi² for the world-wide tropospheric fallout from Operation Redwing and 1 mc/mi² for stratospheric fallout, we would estimate at present that a total of about 22 mc/mi² or Sr^{90} is to be found in the soils of the midwestern United States and that perhaps 15–17 mc/mi² is the total to be expected for similar latitudes elsewhere in the world, the difference being due to our proximity to our own weapons testing site in Nevada. These 22 mc/mi² of Sr^{90} in the soil of the United States amount to about 0.040 MPC units in the top two inches of soil, where most of the fallout is absorbed.

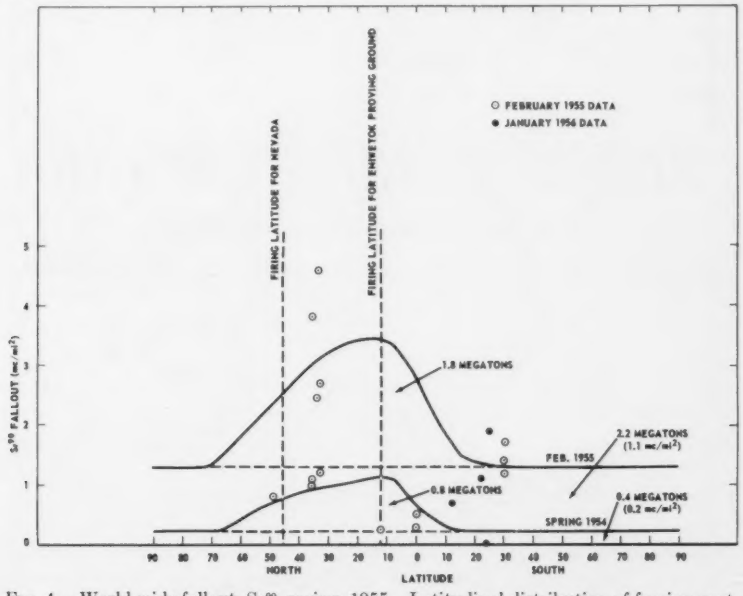

FIG. 4.—World-wide fallout, Sr^{90}, spring, 1955. Latitudinal distribution of foreign post-Castle Sr^{90} fallout (soil assays).

As of the present time, considering the latest human-bone and milkshed data, both domestic and foreign, together with the total fallout figures for corresponding periods, we find that the level of somewhat less than 0.001 MPC units now found in the bones of young children is to be compared with a total Sr^{90} fallout in the soil of about 12 times higher concentration. Additionally, laboratory data have shown that there is a threefold discrimination against strontium as compared to calcium in the assimilation by plants from the soil and that a further factor exists of about

eightfold discrimination against strontium relative to calcium in the excretion of strontium in milk as compared to the cow feed. Earlier,[2] it seemed reasonable to conclude that the human-body burden of Sr^{90} might well be as high as 70 per cent of the concentration in the top soil on which people live. The further evidence just

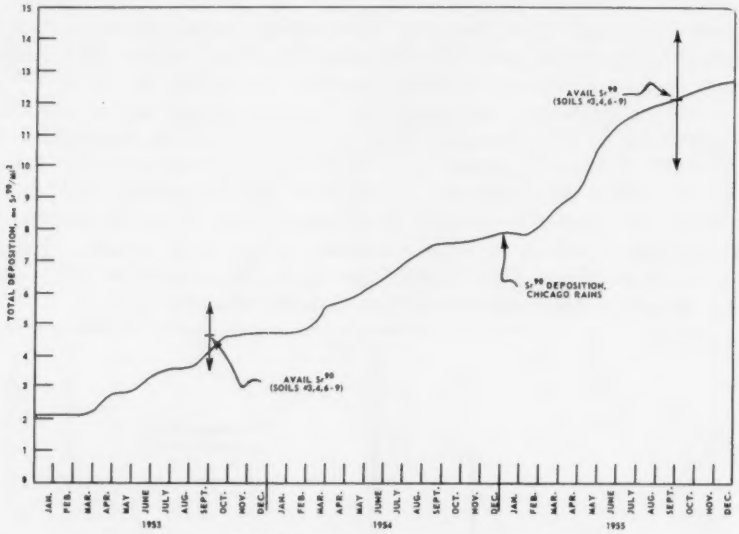

FIG. 5.—Sr^{90} fallout history, Chicago milkshed area

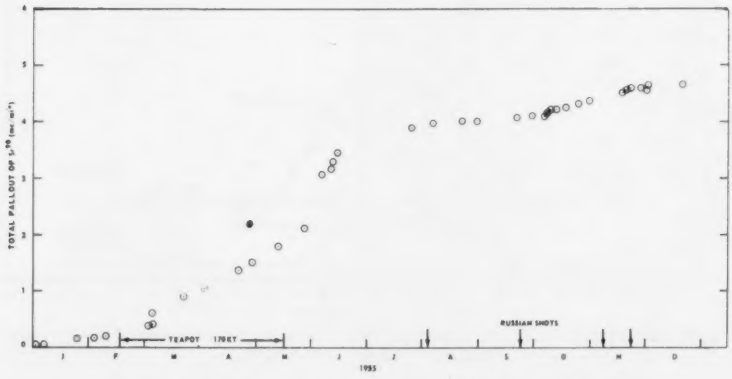

FIG. 6.—Total Sr^{90} fallout in Chicago in 1955 (Pot collection)

cited seems to indicate that this figure is much too high and possibly should be reduced to about 10 per cent. A strict application of the two discrimination factors described would give 4 per cent. Leaf retention of fallout which bypasses the soil causes the figure to be higher. Therefore, at the moment, we would expect that the body burden for children born now in America eventually would amount to be-

tween 0.004 MPC units, corresponding to 10 per cent of the top-soil concentration, and possibly a figure two or three times higher. The stratospheric deposition would be expected to continue at the expected rate, which at the present is about 1.2 mc/yr, so that some fifteen years from now, in the early 1970's a maximum additional total stratospheric fallout of about 6 mc/mi² will have occurred. In the meantime, the present 22 mc/mi² would have been reduced to 15 by radioactive decay, just about compensating for the stratospheric deposition. Thus the conclusion that the body burden in the United States from weapons fired to date would be about 0.004 MPC units, or possibly as high as 0.010 MPC units, seems justified. This level probably will not be exceeded in other countries unless particular factors of environment intervene, since the United States probably has the highest total fallout in the world. It seems very unlikely, however, that environmental factors could increase the level over the United States by more than a factor of 2 or 3.

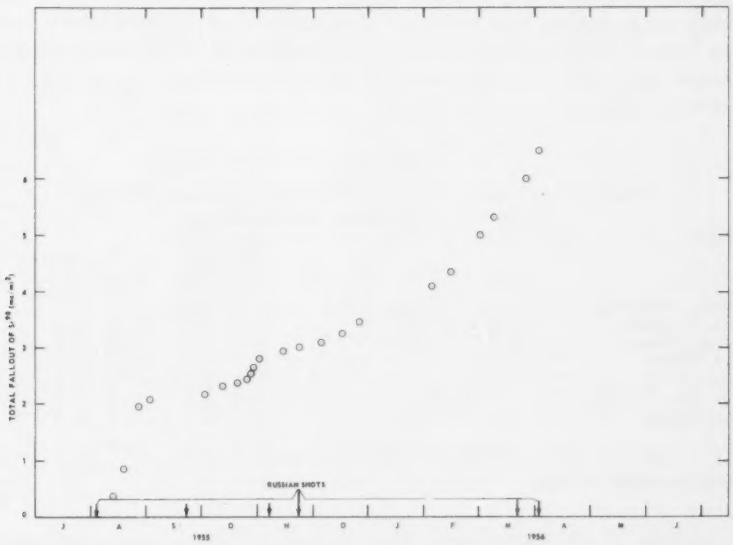

FIG. 7.—Total Sr^{90} fallout in Pittsburgh in 1955–1956 (Pot collection)

B. *Effect of Rainfall.*—As mentioned earlier, there is excellent reason to suppose that the deposition from the troposphere on the earth's surface is best accomplished by rain. By rain is meant not heavy rain but anything which involves the settling of water droplets. This might include fog or mist. The suggestion has been made[3] that the small size of the stratospheric fallout particles gives them a very high mobility due to molecular motion, since, in fact, they probably are almost molecular in dimensions. This high mobility of the particles makes it probable that direct contact of the fallout particles with water droplets will occur. One imagines on this theory that the tiny particles pass through the tropopause from the stratosphere and then meet water droplets in a cloud or mist or rain in the course of their rapid random motion due to collisions with the air molecules. Thus,

rather than the classical Langmuir mechanism of the rain sweeping out the air through which it falls by colliding with the particles themselves, the particles probably collide with the water droplets either before or during the rainstorm, probably most importantly before. It is clear from this mechanism that fog and mist may well be very effective and that a cloud probably gathers a considerable fraction of the fallout from the air in its bulk.

In any case, some experimental evidence has been found for the effect of rainfall on fallout by studying three particularly arid regions: the Imperial Valley in California at the town of Brawley and the western coast of South America at Antofagasta, Chile, and Lima, Peru. The soil in Brawley was sampled in January of this year and found to contain less than 0.6 mc/mi² of Sr^{90}. In order to realize the significance of this number, one should recall that we would have expected about 13 mc/mi² as an average figure for the United States. It is true that a considerable part of this is from the Nevada tests—the depositions of which occur mainly in an easterly direction and might well miss southern California—but it certainly seems that at least 8 mc/mi² would have been expected in the Imperial Valley under normal conditions such as prevail elsewhere in the United States and in Europe and Asia. Thus the observed fallout is not over a few per cent of that expected normally.

TABLE 1

BIOSPHERE Sr^{90} ASSAY, WISCONSIN MILKSHED, SEPTEMBER 30, 1955
(0.001 MPC Units unless Otherwise Stated)

	Soil		Total Sr^{90} (Mc/Mi²)		Alfalfa	
			1955	1953	1955	1953
Farm	0″–2″	2″–6″	Value	Value	Value	Value
Holcomb, Wisconsin	26.7 ± 1.0	3.8 ± 0.14	14.8	5.1	19.2 ± 1.0	8.3
Premo, Wisconsin*	15.0 ± 0.5 (0″–6″)		10.6	3.8	25.5 ± 1.3	4.1
Kurpeski, Illinois*	12.2 ± 0.4 (0″–6.5″)		10.4	4.0	7.05 ± 0.33	7.4
Austin, Illinois	49.9 ± 1.3	9.6 ± 0.4	16.5	4.7	38.0 ± 2.0	5.0
McKee, Illinois	9.8 ± 0.4	0.99 ± 0.04	10.6	6.3	30.5 ± 1.7	14.8
Van Winkle, Illinois	65.1 ± 2.6	10.0 ± 0.4	9.4	3.8	5.76 ± 0.29	5.0
Carver, Illinois	64.5 ± 1.3	14.0 ± 0.7	8.9	3.3	2.73 ± 0.18	2.3
	Average Sr^{90} (mc/mi²)		12.0	4.5		

* Had been plowed to 6″ depth.

In addition to the soil samples, tests were made on the vegetation grown at Brawley, California, as well. As expected, it was found that the level was lower. Lettuce samples collected from this region at the same time as the soil samples showed 0.0004 MPC; broccoli, 0.00025 MPC; green peas, 0.00134 MPC; alfalfa, 0.0021 MPC. These values all are much lower than those for the midwestern United States (shown in Table 2).

The rainfall data for Brawley are as follows: In 1955 the rainfall totaled 1.70 inches, 1.3 inches having occurred in January of that year and 9 months having had no registered rainfall at all. In 1953 the annual total rainfall was only a trace, this trace having occurred in February.

At Antofagasta, Chile, where it has never been known to rain, except possibly on one occasion, we find 0.02 mc/mi² of Sr^{90} in January, 1956, when the general deposition for this latitude was apparently a little over 2 mc/mi.² In other words

about 1 per cent of the fallout expected was found, and 0.02 is hardly larger than the experimental error of measurement.

TABLE 2

1955 DOMESTIC SOIL SAMPLES

Location	Lab No.	Date Sample Taken	Depth of Sample (Inches)	Ca Extracted (Electrodialysis or NH_4Ac)	Calc. Ca (Gm/Ft²)	Sr^{90} Content (0.001 MPC)	Total Sr^{90} (Mc/Mi²)	
Winnebago Co., Ill., Swanson Farm, Site No. 3, Carrington silt loam	551503	9/30/55	0–8	NH_4Ac	82.8	6.83 ± 0.08	9.8	
Rock Co., Wis., Holcomb Farm, Site No. 4, Carrington silt loam	551500	9/30/55	0–2	NH_4Ac	15.0	26.7 ± 1.0	11.4	14.8
Rock Co., Wis., Holcomb Farm, site No. 4, Carrington silt loam	551501	9/30/55	2–6	NH_4Ac	31.8	3.81 ± 0.14	3.4	
Columbia Co., Wis. Premo Farm, Site No. 6, Miami silt loam	551502	9/30/55	0–6	NH_4Ac	25.5	15.0 ± 0.5	10.6	
McHenry Co., Ill., Kurpeski Farm, Site, No. 7	551496	9/30/55	0–6.5	NH_4Ac	30.9	12.2 ± 0.4	10.4	
McHenry Co., Ill., Austin Farm, Site No. 8, Miami silt loam	551504	9/30/55	0–2	NH_4Ac	6.98	49.9 ± 1.3	12.0	16.5
McHenry Co., Ill., Austin Farm, Site No. 8, Miami silt loam	551505	9/30/55	2–6	NH_4Ac	7.8	9.6 ± 0.4	4.5	
McHenry Co., Ill., McKee Farm, Site No. 9, Drummer silt-clay loam	551448	9/30/55	0–2	NH_4Ac	31.2	9.8 ± 0.4	8.5	10.6
McHenry Co., Ill., McKee Farm, Site No. 9, Drummer silt-clay loam	551499	9/30/55	2–6	NH_4Ac	78.4	0.99 ± 0.04	2.1	
Will Co., Ill., Van Winkle Farm, Site No. 11, Plainfield sand	551508	9/30/55	0–2	NH_4Ac	4.4	65.1 ± 2.6	8.0	9.4
Will Co., Ill., Van Winkle Farm, Site No. 11, Plainfield sand	551509	9/30/55	2–6	NH_4Ac	5.1	10.0 ± 0.4	1.4	
Will Co., Ill., Carver Farm, Site No. 12, Plainfield sand	551510	9/30/55	0–2	NH_4Ac	3.7	64.5 ± 1.3	6.7	8.9
Will Co., Ill., Carver Farm, Site No. 12, Plainfield sand	551511	9/30/55	2–6	NH_4Ac	5.6	14.0 ± 0.7	2.2	
Brawley, Calif. (in Imperial Valley)	56316	1/5/56	0–6	Electrodialysis	54.9	≤0.4	≤0.6	

Annual rainfall = 2.57 inches

In Lima, Peru, the total fallout of Sr^{90} in January, 1956, was 0.7 mc/mi². The annual precipitation in Lima averages only 1.89 inches, though there is a considerable amount of ground fog and mist.

It seems clear from these results and the reasonableness of the mechanism of deposition advanced by Mr. Greenfield that there is valid reason to believe that world-wide fallout is small in the absence of precipitation. Also, it is clear from this mechanism that the fallout should not be strictly proportional to total rainfall. Frequent light rains or mists would be expected to be more efficient than occasional heavy rains. The importance of rain is only to be revealed by a study of desert areas and a careful investigation of the scavenging mechanism itself. This work may well prove to be of considerable importance in meteorology, as well as in fallout studies. One should note that the local fallout due to larger particles which descend in the first hours probably does not need rain to precipitate it and occurs in the absence of the precipitation of moisture, although rain may well be able to increase even this fallout.

TABLE 3

Sr^{90} CONTENT OF FOREIGN SOILS AFTER CASTLE

Lat./Long.	Location	Lab. No.	Date Sample Taken	Depth of Sample (Inches)	Sunshine Units	Calc. Exch. Ca (gm/Ft²) (Ammonium Acetate)	(Electrodialysis)	Total Sr^{90} (Mc/Mi²)
33° N/36° E.	Damascus, Syria	55590	2/55	0–4	1.40 ± 0.10	62.8	31.7	1.2
49° N/2° E.	Paris, France	55614	2/55	0–4	0.69 ± 0.05	66.2	41.6	0.8
36° N/139° E.	Tokyo, Japan	55644	2/55	0–4	5.86 ± 0.29	18.9	6.1	1.0
0°/27° W.	Dakar, French W. Africa	55645	2/55	0–4	3.71 ± 0.14	4.4	4.5	0.5
0°/27° W.	Dakar, French W. Africa	55646	2/55	0–4	9.31 ± 0.74	1.3	1.1	0.3
36° N/2° E.	Algiers, Algeria	55647	2/55	0–4	1.20 ± 0.08	60.0	32.6	1.1
36° N/2° E.	Algiers, Algeria	55648	2/55	0–4	2.90 ± 0.20	53.8	47.1	3.8
25° S/57° W.	Asunción, Paraguay	56450	1/56	0–6	11.3 ± 0.75	6.1	6.2	2.0
22° S/46° W.	São Paulo, Brazil	56448	1/56	0–6	3.04 ± 0.27	21.0	14.2	1.2
30° S/30° E.	Durban, Natal, S. Africa	55777	2/55	0–4	4.43 ± 0.19	12.6	13.7	1.7
30° S/30° E.	Durban, Natal, S. Africa	55778	2/55	0–4	15.0 ± 0.08	3.3	3.4	1.4
12° N/45° E.	Aden, Saudi Arabia*	55787	2/55	0–4	0.69 ± 0.09	107.2	12.5	0.24
33° N/40° E.	Ankara, Turkey	55878	2/55	0–4	1.9	96.6	51.6	2.7
34° N/36° E.	Beirut, Lebanon	55591	2/55	0–4	3.2	63.9	52.6	4.6
34° N/36° E.	Terbol, Lebanon	55592	2/55	0–4	1.8	110.7	46.9	2.4
24° S/70° W.	Antofagasta, Chile	56447	1/56	0–½	0.44 ± 0.04	5.2	1.7	0.02
12° S/80° W.	Lima, Peru	56456	1/56	0–6	0.60 ± 0.04	63.9	42.7	0.7
30° S/115° E.	Perth, Australia	55839	2/55	0–4	14.7 ± 1.1	2.8	3.0	1.2

* Exchangeable Ca by isotopic exchange method for this sample was 19.0 gm/ft².

The importance of precipitation as a scavenging mechanism raises the possibility that different regions will be subjected to varying intensities of fallout, depending upon the weather conditions. It will be important to test whether this is so and whether it is a major effect in populated areas. We have evidence showing that extreme aridity greatly reduces the long-range or world-wide fallout, as explained above. The evidence to date does not indicate that it is a major effect for normal climates, in the sense that it does not appear to amount to more than a factor of 2. Regions in which people live normally have enough precipitation so that differences in precipitation appear not to affect the fallout by more than such a factor. Careful study of the data appended and the previously released data[2, 4, 5]

have failed to reveal any more serious deviation. To summarize, desert regions with little or no precipitation, or with only very minimum precipitation, apparently have minimum long-range, world-wide fallout; but other regions do not show that the fallout is proportional to the total precipitation, nor should it be expected to be so; however, detailed conditions related to frequency of precipitation might well be important. The data to date do not reveal deviations from the general average by more than about a factor of 2, and, in fact, they seem to indicate a smaller deviation than this. There is some evidence that certain areas have had more fallout than one might expect on the model described above and on previous occasions. In particular, there are reports that certain areas in England show higher levels, but the deviations appear to be considerably less than twofold.

More important, probably, than the variations in total fallout due to weather conditions is the effect of calcium in the soil in reducing the rate of assimilation of radioactive strontium by plants. The plants assimilate strontium because it is chemically similar to calcium and, since their appetite for calcium is limited, a larger calcium content in the soil dilutes the strontium so that a smaller fraction is assimilated. This effect might amount to a factor of 5 in the human-body assimilation of radiostrontium in regions with very low calcium content in the top soil.

C. *Direct Measurement of the Stratospheric Fallout Content.*—Direct measurement by means of high-flying balloons has shown that the stratosphere does indeed have about the fallout anticipated in it.[6] In addition, it has been found in these measurements that the radiocesium, Cs^{137}, occurs at about the same level as Sr^{90} in millicurie units, indicating that there has been no serious fractionation of the two fission products by the fallout mechanism. It further points out that the sampling of the stratosphere is a practical matter and that measurements can be made of the stratospheric content of radioactive fallout. Such data should greatly assist the whole study of fallout.

D. *Radiocesium Assays for the Biosphere.*—Recently a technique for measuring Cs^{137} in biosphere samples, particularly in human bodies, has been developed by Mr. L. D. Marinelli, of the Argonne National Laboratory.[7] It has been found that about four-millionths of one millicurie is present in an average adult. This corresponds rather well to the expected amount, considering the short residence time of radiocesium in the body, which is about 3 months, and the expected precipitation rate, which is taken to be equal, as a first approximation, to that of radiostrontium. The radiocesium, of course, constitutes no hazard, amounting in radiation dosage to a small fraction of the amount present in the blood in the form of that received from the ordinary potassium present in the body. Potassium is naturally radioactive. It is interesting further evidence, however, that the general model of fallout set forth is consistent with the data and essentially correct.

III. PLANS

It is clear that the peoples of the world are extremely interested in radioactive fallout because of the bearing that the new phenomenology of the nuclear age has on everyone's life. For this reason we must understand radioactive fallout in all its intricacies. It is to be hoped that the study will be a co-operative, international one. The United Nations Scientific Committee on Effects of Atomic Radiation offers an ideal forum for the discussion and consideration of the problem. From

these deliberations will come further suggestions, ideas, appraisals, and statement of the problem. The methods developed in this country for measurement and all the data collected are available to everyone. It is our hope and intention that this problem, like others of the atomic age, will come to be generally understood.

Fallout is normally considered an aspect of atomic warfare and nuclear armament. There is some similarity, however, between the weapons fallout and the hazard from a reactor accident, in which radioactive products would be disseminated over a limited area but would never reach the stratosphere or undergo anything like the world-wide tropospheric dissemination. As it has so often been observed in the past, so it is again true in this instance, that a new fact of nature is likely to have its beneficent as well as its somber and frightening aspects. As we learn about the way the world-wide fallout particle, probably as tiny as a virus molecule, wends its way from the stratosphere through the tropopause into the troposphere and, within a few weeks, collides with a water droplet and thus is brought to the earth's surface by rain, we shall learn more about the circulation of the atmosphere, about the way in which rain is formed, and about the questions which will naturally arise more and more frequently as the world's population increases world-wide pollution of the atmosphere not only with fission products but with the other by-products of our new technological age.

[1] W. W. Kellogg, R. R. Rapp, and S. M. Greenfield, "Close-in Fallout," P-822-AEC, March 12, 1956.

[2] W. F. Libby, "Radioactive Strontium Fallout," these PROCEEDINGS, **42**, 365, 1956.

[3] Stanley Greenfield, Rand Corporation.

[4] John H. Harley, Edward P. Hardy, Jr., George A. Welford, Ira B. Whitney, and Merril Eisenbud, "Summary of Analytical Results from the HASL Strontium Program to June 1956," NYO-4751, August 31, 1956.

[5] Project Sunshine Bulletins Nos. 11 and 12, Enrico Fermi Institute for Nuclear Studies University of Chicago, December 1, 1955, and August 1, 1956.

[6] A. P. Hardy and S. Tarras, "General Mills High Altitude Balloon Filter Samples," Memorandum, New York Operations Office, July 2, 1956.

[7] "Gamma-Ray Activity of Contemporary Man," *Science*, **124**, 122, July 20, 1956.

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*AUDITORY RESPONSES IN THE COMMON BOX TURTLE**

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We have previously reported observations of inner-ear potentials in response to sound in three species of turtles, *Chrysemys picta picta*, *Pseudemys scripta*, and *Clemmys insculpta*.[1] Sensitivity curves were obtained for these ears by measuring the sound pressure necessary to produce a standard amount of potential, and these curves were found to be similar in form for all three species. The sensitivity is rather uniformly keen for tones between 100 and 700 cycles and then grows progressively poorer for the higher tones up to 3,000 cycles, beyond which a response cannot be obtained at any intensity that is safe to apply to the ear.

We have now extended our observations to a fourth species, the common box turtle *Terrapene carolina carolina*. This species, like the others mentioned, belongs to the extensive family of Emydidae, but anatomically it presents a number of variations from the general type. Of particular interest are the variations in the form of the skull in the region of the ear. Also, this turtle is relatively small, rarely exceeding 5 inches in length of carapace. Our specimens were mostly adult and varied from 4 to 5 inches in carapace length.

The observations reported here were made on eight animals, and in four of these on both ears. In addition, a number of animals were tested which on subsequent examination were found to be suffering from middle-ear infections. In these the sensitivity was 20–40 db. poorer than in healthy animals.

The experimental procedure was similar to the one used principally in our earlier studies. As before, the active electrode was a steel needle, insulated except at the tip, which was inserted into the perilymph space of the ear. However, the placement of this electrode was considerably more difficult than in the other species. The bones of the temporal region of the skull are relatively thick in this species, and usually we were unable to find the sutures that in the other forms served as reliable landmarks for the location of inner-ear structures. The most satisfactory procedure was first to remove the temporal muscle and then to open the posterior part of the middle-ear cavity (a procedure that itself has no significant effect upon the sensitivity).[2] Thereupon two features are revealed: a dark area that represents the shadow of the cavernous sinus lying deep beyond the inner wall of the middle-ear cavity and a translucent area that marks the location of the posterior portion of the pericapsular recess. With the aid of these points a hole was drilled through the bone in the region of the posterior semicircular canal, and the needle electrode was tightly fitted into this hole to prevent the escape of perilymph. An indifferent electrode was inserted beneath the skin of the head.

Tones were applied to the ear through a rubber tube sealed over the tympanic membrane, and observations were made of the sound pressures necessary to produce a standard potential of 0.3 microvolt. Results for two of the animals are shown in Figure 1. Here the solid line represents one of the more sensitive animals and the broken line one of the less sensitive. These curves follow the form already described for the other species of turtles, showing good sensitivity over a range from 100 to 600 cycles and progressively poorer sensitivity for higher and lower tones. As observed earlier, it usually was not possible to obtain measurable potentials for tones above 3,000 cycles at intensities that were safe to apply to the ear. The auditory sensitivity of this species, as measured by the inner-ear potentials, is closely similar to that of *Chrysemys picta picta* and *P. scripta* and somewhat below that of *Clemmys insculpta* in the most favorable region of 100–600 cycles.

Some of our observations have dealt with the mode of action of the tympanic membrane. This membrane consists of a cartilagenous disk (the extracolumellar disk) lying beneath the largely undifferentiated skin at the side of the head and attached to this skin by a loose network of fibrous and fatty tissue. The attachment of the disk to the edges of the tympanic opening in the skull is very loose except at the posterior edge, where there is a heavy ligament, the posterior ligament, connecting to the quadrate and squamous bones. A gross displacement of the tympanic membrane causes the extracolumellar disk to move as on a hinge at the posterior

ligament. This motion is transmitted through the medial process of the extracolumella to the columella, which is a thin bony rod leading inward to the otic capsule, where it expands to a funnel-like stapes lying in the oval window. The rocking motion of the tympanic membrane produces a piston-like motion of the stapes in the oval window. The question arises whether this rotational mode of movement of the tympanic membrane holds also for the much smaller displacements produced by sounds.

To study this movement, we set the tympanic membrane into vibration by applying to its surface the point of a needle that was driven by a piezoelectric crystal. The crystal vibrator was constructed so as to have great stiffness in the direction of the vibrations and so that the moderate loading involved in this application had only a negligible effect upon the vibratory amplitude. The inner-ear potentials produced by driving the tympanic membrane in this manner were observed as the needle was applied at various points over the membrane surface. A heavy manipulator with graduated scales was used to hold the vibrator and to control its placement. Care was taken to restrict the driving amplitudes to the linear range of operation of the ear.

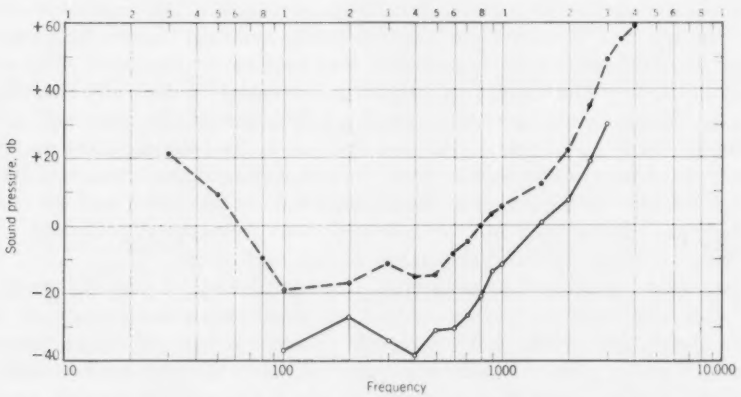

FIG. 1.—Sensitivity curves for two turtles of the species *Terrapene carolina carolina*. The sound pressure is represented in decibels relative to 1 dyne per square centimeter.

The results of a typical experiment are shown in Figure 2. Here the vibrating needle was first applied at a point somewhat in front of the anterior edge of the extracolumeler disk and then was moved posteriorly by small steps until it was well behind the posterior edge of this disk. At every position the driving current for the crystal was adjusted to produce a constant inner-ear potential of 0.3 microvolt. The result was a curve, as shown, containing sharp breaks at the two ends. Subsequent measurements, made after removing the superficial skin layer, proved that these breaks appeared as the driving point was moved over the anterior and posterior edges of the extracolumellar disk and onto the neighboring tissues, which then communicated the motions to the columellar system in a complicated way. As may be seen, the vibratory amplitude necessary to produce the standard response fell off in a systematic manner as the driving point was moved along the anteroposterior dimension of the disk. If the position on the extreme anterior edge is taken as a reference (0 db.), as indicated in the figure, the necessary amplitude falls

to a value of 40.7 db. less at the extreme posterior edge of the disk. This observation means that the lever ratio of the system is continually changing along this dimension, and the driving point is being moved toward a fulcrum located at the posterior end. It follows that the normal action of the tympanic membrane in response to sound is a rocking motion about the posterior ligament. It is of interest to note that a similar motion of this membrane occurs in higher animals, including man, as best shown by the observations of Békésy.[3]

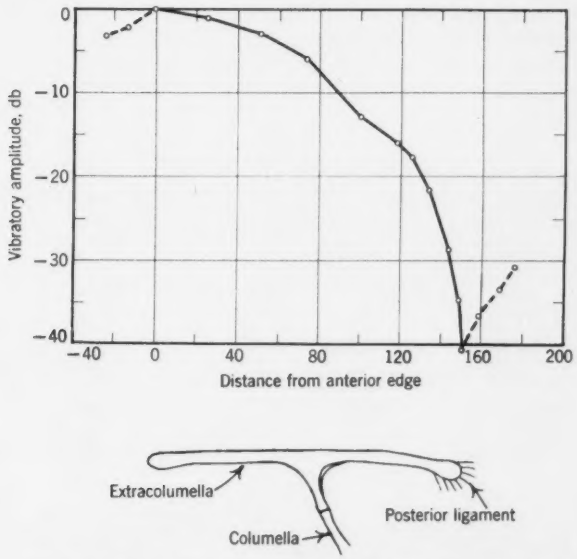

FIG. 2.—Relative amplitude of vibration of the tympanic membrane necessary to produce a standard response of 0.3 microvolt when the driving point was varied along the antero-posterior axis. This amplitude is shown in decibels relative to the least effective position at the anterior edge of the extracolumellar disk. Distances from this position are shown on the abscissa in thousandths of an inch. Below the graph a cross-sectional view of the extracolumellar disk is represented on the same scale as the abscissa.

Summary.—The auditory sensitivity of the common box turtle, *T. carolina carolina*, was measured in terms of the inner-ear potentials. This ear resembles that of other species of turtles studied previously in that it shows keen sensitivity in the region of 100–600 cycles and progressively poor sensitivity for higher and lower tones. The tympanic membrane was found to undergo a rotational mode of movement in response to applied forces, swinging about a hinge formed by its posterior ligament.

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[1] E. G. Wever and J. A. Vernon, "The Sensitivity of the Turtle's Ear as Shown by Its Electrical Potentials," these PROCEEDINGS, **42**, 213–220, 1956.

[2] E. G. Wever and J. A. Vernon, "Sound Transmission in the Turtle's Ear," *ibid.*, pp. 292–299.

[3] G. von Békésy, "Über die Messung der Schwingungsamplitude der Gehörknochelchen mittels einer kapazitiven Sonde," *Akust. Z.*, **6**, 1–16, 1941.

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CONTENTS

Page

ASTRONOMY.—THE COSMIC ABUNDANCES OF POTASSIUM, URANIUM, AND THORIUM AND THE HEAT BALANCES OF THE EARTH, THE MOON, AND MARS . . . *By Harold C. Urey* 889

BACTERIOLOGY.—THE ISOLATION OF A NEW VIRUS ASSOCIATED WITH RESPIRATORY CLINICAL DISEASE IN HUMANS . . . . . *By Winston H. Price* 892

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BIOCHEMISTRY.—ON THE FUNCTIONS OF X-IRRADIATED "FEEDER" CELLS IN SUPPORTING GROWTH OF SINGLE MAMMALIAN CELLS . *By Harold W. Fisher and Theodore T. Puck* 900

BOTANY.—THE PRODUCTION OF TUMOROUS ABNORMALITIES IN FERN PROTHALLI BY IONIZING RADIATIONS . . . . . *By Carl R. Partanen and Taylor A. Steeves* 906

CHEMISTRY.—A NEW METHOD FOR THE MEASUREMENT OF SELF-DIFFUSION . . . . . *By Henry C. Thomas* 909

MATHEMATICS.—PARABOLIC SYSTEMS OF DIFFERENTIAL EQUATIONS WITH TIME-DEPENDENT COEFFICIENTS . . . . . *By Felix E. Browder* 914

MATHEMATICS.—ON ABSOLUTE CONVERGENCE OF FOURIER SERIES . *By Fu Cheng Hsiang* 917

MATHEMATICS.—A SEQUENTIAL DECISION PROBLEM WITH A FINITE MEMORY . . . . . *By Herbert Robbins* 920

MATHEMATICS.—SLIP SURFACES IN PLASTIC SOLIDS. . . . . *By T. Y. Thomas* 923

MATHEMATICS.—NOTE ON DEGREE OF APPROXIMATION TO ANALYTIC FUNCTIONS BY RATIONAL FUNCTIONS WITH PREASSIGNED POLES . . . . . *By J. L. Walsh* 927

PHYSICS.—SIMPLIFIED MODEL TO DEMONSTRATE THE ENERGY FLOW AND FORMATION OF TRAVELING WAVES SIMILAR TO THOSE FOUND IN THE COCHLEA . *By G. von Békésy* 930

PHYSICS.—CURRENT RESEARCH FINDINGS ON RADIOACTIVE FALLOUT *By Willard F. Libby* 945

PSYCHOLOGY.—AUDITORY RESPONSES IN THE COMMON BOX TURTLE. . . . . *By Ernest Glen Wever and Jack A. Vernon* 962

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